Gauged Ghost Condensation and Spontaneous Lorentz-Violation at High Scales

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1 Introduction

All experimental evidence to date suggests that Lorentz-invariance is a good symmetry of the universe. In some sense, the best evidence that Lorentz-invariance is not broken is the phenomenal success of General Relativity. Diffeomorphisms (*i.e.* local Lorentz transformations) protect the graviton from picking up a mass, so if Lorentz-invariance were either explicitly or spontaneously broken then we would expect the graviton to acquire additional "longitudinal" polarizations. The fact that we have not observed gross violations of Newton's Law — along with the stunning success of relativistic particle physics — gives us excellent evidence that Lorentz-invariance is preserved over a wide range of energy scales.

In this light, the theory of ghost condensation [1, 2] seems to both confirm and contradict the notion that it is hard to build an experimentally viable effective field theory that breaks Lorentz-invariance. The ghost condensate spontaneously breaks time diffeomorphisms at an energy scale M, and below this energy there is a Goldstone boson that non-linearly realizes the broken symmetry. This Goldstone mode mixes with gravity and at linearized level modifies gravity at length scales greater than $r_c = M_{\text{Pl}}/M^2$ and at time scales longer than $t_c = M_{\text{Pl}}^2/M^3$. If we demand no modification to Newton's Law over the lifetime of the universe, then we are forced to take $M < 10$ MeV. Therefore, while it is indeed possible to have a viable, consistent theory with spontaneously broken Lorentz-invariance, the scale of Lorentz-violations in the example of ghost condensation is very low from the point of view of particle physics.

Of course, this does not mean that there are no interesting experimental signals if time diffeomorphisms are broken at low energy scales. As we argue in [3], the Goldstone boson associated with broken time diffeomorphisms can mediate a Lorentz-violating inverse-square law force between spins, and this force may be strong enough to observe at millimeter length scales. Also, there are non-linear gravitational effects that become important even around modest gravitational sources [4], and depending on the structure of the non-linear interactions in the theory, the scale M could be as small as y eV or as large as x GeV.

Still, we might ask whether there exists an experimentally viable effective field theory that naturally incorporates spontaneous Lorentz-violations at high energy scales without relying on non-linear effects. In a sense, this question is motivated by the idea that Lorentz-symmetry could be softly violated by unknown physics at the Planck scale [ref?]. In certain string theory constructions, one finds non-zero vacuum expectation values for vectors and tensors at energy scales close to but less than M_{Pl} [ref?]. From the point of view of effective field theory, it is a bit of a mystery how accelerator-scale Lorentz-symmetry is protected from Planck-scale Lorentz-violations [ref?]. In this paper, we will show in effective field theory language that it is indeed possible to safely break Lorentz-symmetry at a scale z GeV with minimal effect on both gravitational and Standard Model measurements.

The idea we want to pursue in this paper is that there are a number of ways of spontaneously breaking Lorentz-symmetry, and the choice of which — if any — breaking mechanism to use is ultimately an experimental question. Because rotational invariance is very well tested, we expect that Lorentz-invariance is broken at most to $SO(3)$-invariance. In the theory of ghost condensation, the assumption is that time diffeomorphisms are spontaneously broken but space diffeomorphisms are preserved. If we only care about maintaining $SO(3)$-invariance, then we should also consider the possibility that time diffeomorphisms are preserved but space diffeomorphisms are spontaneously broken. This yields a different (and seemingly unhealthy) low energy effective theory with different experimental signatures than ghost condensation, and if we were ever to observe Lorentz-violations, we could compare the two models to see which one better matches measurements.

In this paper, we look at a theory where time diffeomorphisms and a new $U(1)$ gauge symmetry are spontaneously broken down to a diagonal $U(1)$ gauge group. For obvious reasons, we (lamely) call this the theory of gauged ghost condensation. It may seem a bit strange to consider the diagonal subgroup composed of a (non-compact) space-time symmetry and a (compact) internal symmetry, and while there may be topological problems with this construction, we will see that it is straightforward to generate the local low-energy effective Lagrangian that describes this symmetry breaking pattern.

We will also see that for large enough gauge coupling, this theory has some distinct advantages over "ungauged" ghost condensation. In the ungauged case, the Goldstone boson associated with broken time diffeomorphisms develops a Jeans-like instability due to mixing with gravity. While this instability is softened by non-linear effects in the ungauged theory, the instability is completely removed in the gauged theory for sufficiently large gauge coupling. Also, for large enough gauge coupling, the modification of Newton's law due to mixing with the Goldstone boson is virtually elliminated, allowing us to raise the scale of spontaneous Lorentz-breaking high above the electroweak scale.

There is a qualitative way to understand why our theory is less dangerous for larger gauge coupling. If the gauge coupling is zero, then there is a unitary gauge where the Goldstone boson is completely "eaten" by the graviton. Similarly, if gravity is turned off, then there is a unitary gauge where the Goldstone boson is completely eaten by the $U(1)$ gauge boson. In between these two extremes, the Goldstone boson can be "more eaten" or "less eaten" by the graviton depending on the value of the gauge coupling. Because the graviton universally couples to matter but the $U(1)$ gauge boson can safely live in a hidden sector, we can reduce the observable effects of Lorentz-violations simply by increasing the gauge coupling. This effectively hides Lorentz-violations from the gravity sector and the Standard Model, assuming no Standard Model fields are charged under the new $U(1)$ gauge group.

In short, we have a viable, consistent, low energy effective theory of spontaneously broken Lorentz-invariance that reduces to ghost condensation in the limit of zero gauge coupling. For modest values of the gauge coupling, we can raise the symmetry breaking scale as high as $M = z$ GeV with minimal experimental constraints. Newton's law is slightly modified through mixing with the gauge boson and Goldstone boson but the modification is so weak as to be virtually nonexistent. The primary constraints on this theory seem to come from higher dimension couplings to the Standard Model that are mediated by graviton loops, but using naive dimensional analysis, these couplings appear very suppressed.

2 Gauged Ghost Condensation

The low energy effective Lagrangian from gauged ghost condensation arises in any theory where time diffeomorphisms and a $U(1)$ gauge symmetry are spontaneously broken down to a diagonal $U(1)$ gauge group. It is straightforward, albeit tedious, to find the quadratic piece of the Lagrangian by brute force calculation. If we linearize the metric around flat space, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, then the diffeomorphism generated by $x^\mu \to x^\mu + \xi^\mu$ acts on $h_{\mu\nu}$ to leading order as

$$h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \tag{1}$$

and acts on a vector field A_μ as

$$A_\mu \to A_\mu - \partial_\mu \xi^\nu A_\nu - \xi^\nu \partial_\nu A_\mu. \tag{2}$$

A $U(1)$ gauge symmetry acts on A_μ in the usual way

$$A_\mu \to A_\mu - \partial_\mu \alpha, \tag{3}$$

and we can generate the gauged ghost condensate Lagrangian by writing all terms invariant under the simultaneous transformations ξ^μ and $\alpha = M\xi^0$. For example, the term $(h_{00} - A_0/M)^2$ is invariant to leading order.

A simpler way to derive the gauged ghost condensate Lagrangian is to imagine a charged scalar field getting a vacuum expectation value in a time-like direction. We can imagine this scalar field being a ghost so fluctuations around $\phi = 0$ are unstable. The stabilization (or condensation) of this gauged ghost field spontaneously breaks time diffeomorphisms and the $U(1)$ gauge symmetry down to a diagonal $U(1)$ gauge symmetry. While the underlying physics is really the gauge breaking pattern, we will find it convenient to work in the scalar condensation language. This is analogous to using the language of tachyon condensation (*i.e.* a Higgs field getting a non-zero vev because it has a negative mass-squared) to describe the underlying physics of spontaneous gauge symmetry breaking.

More concretely, consider a $U(1)$ gauge theory and a scalar field ϕ with mass dimension zero that transforms under gauge transformations as

$$\phi \to \phi + \alpha, \qquad A_\mu \to A_\mu - \partial_\mu \alpha. \tag{4}$$

By gauge invariance, the action can only depend on $F_{\mu\nu}$ and $D_\mu\phi$, where

$$D_\mu \phi = \partial_\mu \phi + A_\mu. \tag{5}$$

If the vacuum of this theory satisfies $\langle D_\mu \phi \rangle = 0$, then in the absence of gravity, the leading terms in the Lagrangian take the form

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{v^2}{2} (D_\mu \phi)^2, \tag{6}$$

and we see in unitary gauge ($\phi = 0$) that this is the Lagrangian for a vector field of mass $m = gv$. We would call the field ϕ the Goldstone boson of spontaneous $U(1)$ symmetry breaking.

But what if the vacuum of this theory were not $\langle D_\mu \phi \rangle = 0$? We might imagine that for some (dynamical?) reason, the vacuum of the theory spontaneously broke Lorentz invariance:

$$\langle D_0 \phi \rangle = M. \tag{7}$$

Note that when we turn on gravity, this vacuum indeed breaks time diffeomorphisms and the $U(1)$ down to a diagonal $U(1)$. That is, we can use time diffeomorphisms to align the time coordinate with the field ϕ via $\phi = Mt$, but only if we simultaneously make a gauge transformation such that $\phi = Mt$ is not just pure gauge.

In analogy to the theory of ghost condensation, we can write down the generic theory that is invariant under Lorentz invariance and gauge transformations before spontaneous symmetry breaking. (We assume a $\phi \to -\phi$ symmetry.)

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^2 + M^4 P(X) + (\partial^\mu D_\mu \phi)^2 Q(X) + (\partial_\mu D_\nu \phi)^2 R(X) + \ldots, \qquad X = \frac{D_\mu \phi D^\mu \phi}{M^2}. \tag{8}$$

Because the Lagrangian we have written down is Lorentz-invariant, we can minimally couple gravity in the usual way. If the vacuum really satisfies $\langle D_0 \phi \rangle = M$, then we expect that P will have a minimum at $X = 1$,

$$P(X) = \frac{1}{8}(X-1)^2 + \ldots, \tag{9}$$

but we have no *a priori* reason to expect $Q(X)$ or $R(X)$ to have any particular functional form, and to leading order we will take

$$Q(X) = -\frac{\beta}{2}, \qquad R(X) = \frac{\gamma}{2}, \tag{10}$$

where β and γ are expected to be $\mathcal{O}(1)$ coefficients.

3 Polarizations of Gauged Ghost Excitations

Before turning on gravity, we can look at the Lagrangian for ϕ and A_μ alone. If we go to pseudo-unitary gauge ($\phi = Mt$), then the vacuum of the theory satisfies $\langle A_\mu \rangle = 0$, and

$$X = 1 + \frac{2A_0}{M} + \frac{A_0^2 - A_i^2}{M^2}. \tag{11}$$

The quadratic piece of our effective Lagrangian is thus

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{2} M^2 A_0^2 - \frac{\beta}{2}(\partial^\mu A_\mu)^2 + \frac{\gamma}{2}(\partial_\mu A_\nu)^2. \tag{12}$$

Note that we can set $\gamma = 0$ by adjusting the value of β and the gauge coupling. In the limit that we decouple gravity, A_μ represents the three polarizations of a modified $U(1)$ gauge field (and any possible ghost or tachyons that we haven't forseen). Also, note that each of the terms $\frac{1}{2} M^2 A_0^2$ and $\frac{\beta}{2}(\partial^\mu A_\mu)^2$ are gauge fixing terms taken alone, and we only acquire a physical Goldstone mode if both terms are taken together.

Though a bit obscure in this pseudo-unitary gauge, in the limit $g \to 0$ we can recover our favorite ghostone Lagrangian:

$$\mathcal{L} = \frac{1}{2}\dot{\pi}^2 + \frac{\beta - \gamma}{2M^2}(\nabla^2 \pi)^2. \tag{13}$$

To see this, reintroduce the gauge symmetry in equation (12) by making the transformation $A_\mu \to A_\mu + \partial_\mu \pi$, and then take $A_\mu = 0$ and go to canonical normalization for the π field. Note that we have dropped higher order time derivatives in equation (13), which signal ghosts or tachyons at the scale M. This should be expected, because our effective theory only makes sense below the scale M.

For the moment, we will stay in the pseudo-unitary gauge of equation (12) and find the propagating degrees of freedom of the field A_μ. Later, when we are more interested in the modification of gravity, we will use a different gauge choice, but in pseudo-unitary gauge it is easier to follow our nose and find the polarizations without worrying about gauge fixing conditions.

Starting with equation (12), it is straightforward to derive the equation of motion for A_μ. Without loss of generality, we will work with $\gamma = 0$. We look at plane wave solutions

$$A_\mu = \epsilon_\mu(p) e^{ip\cdot x}, \tag{14}$$

where

$$p_\mu = (\omega(k), \vec{k}), \qquad \epsilon_\mu = (\epsilon_0, \vec{\epsilon}). \tag{15}$$

The A_0 equation of motion looks like

$$(k^2 - \omega^2 g^2 \beta + g^2 M^2)\epsilon_0 = \omega(\vec{k}\cdot\vec{\epsilon})(1 - g^2\beta), \tag{16}$$

and after some simplifications, the A_i equation of motion gives

$$(\omega^2 - k^2)\vec{\epsilon} = \vec{k}(\vec{k}\cdot\vec{\epsilon})(1 - g^2\beta)(Q\omega^2 - 1), \qquad Q = \frac{1 - g^2\beta}{k^2 - \omega^2 g^2\beta + g^2M^2}. \tag{17}$$

Though we can easily solve for the polarization, it is instructive to first consider the special case $\beta = 1/g^2$. If we look at the Lagrangian in this limit

$$\mathcal{L} = \frac{1}{2g^2}(A_\mu \Box A^\mu) + \frac{1}{2}M^2 A_0^2, \tag{18}$$

we immediately recognize that A_i contains three healthy modes, but A_0 is a ghost with mass gM, so our theory makes sense as an effective theory up to the scale gM. For arbitrary β we expect a ghost with mass of order M and three healthy propagating modes.

Taking $\vec{k} = (0, 0, k)$, the two transverse polarizations are

$$\epsilon^\pm_\mu = (0, 1, \pm i, 0) \quad \text{with} \quad \omega^2 = k^2, \tag{19}$$

so despite the fact we have broken Lorentz-invariance, the transverse polarizations still travel at the speed of light. The "longitudinal" mode has

$$\epsilon^3_\mu = (\omega k Q, 0, 0, 1) \quad \text{with} \quad \omega^2 = \frac{M^2 + 2k^2\beta - M\sqrt{M^2 + 4k^2\beta(1 - g^2\beta)}}{2\beta} \simeq g^2\beta k^2 + (1 - g^2\beta)^2 \beta \frac{k^4}{M^2} + \mathcal{O}(k^6). \tag{20}$$

For $g = 0$, we recover the dispersion relation for the ghostone boson, but for non-zero g there is an addition k^2 piece. Note that for $\beta > 1/g^2$, the "longitundinal" polarization travels faster than light and becomes unphysical for

$$k^2 > \frac{M^2}{4\beta(g^2\beta - 1)}. \tag{21}$$

There is also the polarization for the ghost excitation:

$$\begin{aligned} \epsilon_\mu^{\text{ghost}} = (\omega k Q, 0, 0, 1) \qquad \text{with} \qquad \omega^2 &= \frac{M^2 + 2k^2\beta + M\sqrt{M^2 + 4k^2\beta(1 - g^2\beta)}}{2\beta} \\ &\simeq \frac{M^2}{\beta} + (2 - g^2\beta)k^2 - (1 - g^2\beta)^2\beta\frac{k^4}{M^2} + \mathcal{O}(k^6). \end{aligned} \tag{22}$$

Again, note that this ghost excitation has mass of order M, so it does not affect low energy physics.

4 Mixing With Gravity

While pseudo-unitary gauge is convenient for identifying the degrees of freedom, it obscures the simple picture for the modification of gravity due to mixing with the new gauge-ghost sector. Though we expect mixing between A_i and the vector mode of gravity, we are interested mostly in mixing involving the scalar mode of gravity, because the scalar mixing is related to the modification of Newton's Law.

As shown in the original ghost condensate paper, in the non-relativistic limit and in a suitable gauge, the Einstein-Hilbert Lagrangian can be taken to be

$$\mathcal{L}_{\text{EH}} = -M_{\text{Pl}}^2(\nabla\Phi)^2, \tag{23}$$

with

$$h_{00} = 2\Phi, \qquad h_{0i} = 0, \qquad h_{ij} = 2\Phi\delta_{ij}. \tag{24}$$

We also want to focus on the scalar modes of A_μ. If we expand $\phi = Mt + \pi$, then the theory in equation (8) still has a gauge invariance under which π shifts. A general scalar excitation of A_μ can be parameterized as

$$A_0 = \chi', \qquad A_i = \partial_i\eta, \tag{25}$$

but performing the gauge transformation $A_\mu \to A_\mu - \partial_\mu\eta$, we can fix gauge and bring A_μ into the form

$$A_0 = \chi, \qquad A_i = 0. \tag{26}$$

Returning to equation (8), we can minimally couple gravity in the standard way. Kinetic mixing will only occur from terms quadratic in the fields.

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{EH}} - \frac{1}{4g^2}F_{\mu\nu}^2 + \frac{M^4}{8}(X-1)^2 - \frac{\beta}{2}(\partial^\mu D_\mu\phi)^2 + \ldots, \\ &= -M_{\text{Pl}}^2(\nabla\Phi)^2 + \frac{1}{2g^2}(\nabla\chi)^2 + \frac{M^4}{2}\left(\Phi - \frac{\dot\pi + \chi}{M}\right)^2 - \frac{\beta}{2}(\nabla^2\pi)^2, \end{aligned} \tag{27}$$

where we have dropped terms of higher order in time derivatives which signal the breakdown of our theory near the energy scale M. Going to canonical normalization

$$\Phi = \frac{\Phi_c}{M_{\rm Pl}\sqrt{2}}, \qquad \chi = g\chi_c, \qquad \pi = \frac{\pi_c}{M}, \tag{28}$$

the momentum space Lagrangian is

$$\mathcal{L} = \frac{1}{2}\left(\begin{array}{ccc} \pi_c & \chi_c & \Phi_c \end{array}\right)\left(\begin{array}{ccc} \omega^2 - \beta k^4/M^2 & igM\omega & -im\omega \\ -igM\omega & k^2 + g^2M^2 & -gmM \\ im\omega & -gmM & -k^2 + m^2 \end{array}\right)\left(\begin{array}{c} \pi_c \\ \chi_c \\ \Phi_c \end{array}\right), \tag{29}$$

where

$$m = \frac{M^2}{\sqrt{2}M_{\rm Pl}}. \tag{30}$$

We can identify the dispersion relation for the propagating Goldstone mode by setting the determinant of the kinetic matrix to zero.

$$\omega^2 = \beta\left(g^2 - \frac{m^2}{M^2}\right)k^2 + \frac{\beta k^4}{M^2} \tag{31}$$

Immediately, we recognize that for $g > m/M$, the Jeans instability in this theory is removed, and we do not expect any exponentially growing solutions around sources.

It is also straightforward invert the kinetic matrix to find the modification of the $\langle\Phi_c\Phi_c\rangle$ propagator and hence the modification to Newton's law:

$$\frac{-1}{k^2}\left(1 - \frac{m^2\beta k^2}{M^2\omega^2 - \beta k^4 - \beta(g^2M^2 - m^2)k^2}\right) \tag{32}$$

Just as in ghost condensation, to see an $\mathcal{O}(1)$ modification of gravity, we need to be close to on shell for π excitations, but now that the physical π has a dispersion relation that starts as $\omega \sim g\sqrt{\beta}k$, the time it takes to reach steady state at a distance r is roughly

$$t = \frac{r}{g\sqrt{\beta}}. \tag{33}$$

While not necessary, it is certainly conceivable that $g\sqrt{\beta} \sim \mathcal{O}(1)$ such that π waves travel at some reasonable (subluminal) velocity. If this is the case, we can safely pass to the $\omega \to 0$ limit without worrying about time retardation effects. In that limit, we can Fourier transform the $\langle\Phi_c\Phi_c\rangle$ propagator to find the modification to Newton's Law:

$$V(r) = -\frac{G}{r}\frac{1}{1-\epsilon^2}\left(1 - \epsilon^2 e^{-r/r_0}\right) \tag{34}$$

where G is Newton's constant and

$$\epsilon = \frac{m}{gM} = \frac{M}{\sqrt{2}gM_{\rm Pl}}, \qquad r_0 = \frac{1}{\sqrt{g^2M^2 - m^2}} \simeq \frac{1}{gM}\left(1 + \frac{\epsilon^2}{2} + \ldots\right). \tag{35}$$

Note that in order for this potential to make any sense (*i.e.* in order to assure there is no exponentially growing mode from a Jeans instability), ϵ must be less than 1.

5 Couplings to Matter

We can also imagine coupling $D_\mu\phi$ directly to matter. If matter is explicitly charged under the $U(1)$, then there are strong constraints [ref?]. In analogy to the Universal Dynamics paper, we could have a coupling

$$\mathcal{L}_{\rm int} = \frac{1}{F}\bar{\Psi}\gamma^\mu\gamma^5\Psi D_\mu\phi. \tag{36}$$

Leads to explicit L.V. term. Decoupling gravity, working in unitary gauge, and going to non-relativistic limit:

$$\mathcal{L}_{\rm int} = \frac{1}{F}\vec{s}\cdot\vec{A} \tag{37}$$

In the non-relativistic limit, we can simply set $A_0 = 0$. (How do we know we can go to non-relativistic limit? Because π waves travel at $v = g\sqrt{\beta}$. As long as v is fast enough...) In this limit, the $\langle A_i A_j\rangle$ propagator is (un-normalized)

$$\langle A_i A_j\rangle = \frac{1}{k^2}\left(g^2\delta_{ij} - (g^2 - 1/\beta)\frac{k_i k_j}{k^2}\right) \tag{38}$$

Fourier transforming to find the potential between spins $\vec{S}_1$ and $\vec{S}_2$

$$V(r) = \frac{1}{8\pi r}\left((g^2+1/\beta)\vec{S}_1\cdot\vec{S}_2 + (g^2-1/\beta)(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})\right) \tag{39}$$

What about gravity with moving source? Consider the $\langle\Phi\Phi\rangle$ propagator with ω in the $r_0 \to 0$ limit. r_0 is a very small scale, irrelevant for GR questions.

$$\langle\Phi\Phi\rangle = \frac{-1}{k^2} + \frac{\epsilon^2}{\omega^2/v_c^2 - k^2(1-\epsilon^2)} \tag{40}$$

To find moving potential set $\omega = \vec{k}\cdot\vec{v} + i\delta$ where δ sets pole prescription for retarded potential. If $v < v_c(1-\epsilon^2)$ then k poles are always imaginary and we can set $\delta = 0$. Expanding to leading order in v/v_c and ϵ:

$$\langle\Phi\Phi\rangle = -\frac{1}{k^2}\left(1+\epsilon^2+\epsilon^2\frac{v^2}{v_c^2}(\hat{k}\cdot\hat{v})^2\right) \tag{41}$$

Fourier transforming this to find potential between sources:

$$V(r) = -\frac{1}{r}\left(1+\epsilon^2+\frac{\epsilon^2}{2}\frac{v^2}{v_c^2}\left(1-(\hat{r}\cdot\hat{v})^2\right)\right) \tag{42}$$

Which we compare to PPN parameters as Shinji said. Again, formula is only valid for

6 Experimental Constraints

It is clear from the form of equation (34) that for sufficiently large g, it would be very difficult to distinguish between standard GR and a universe filled with a gauged ghost condensate. Though there is a $1/(1-\epsilon^2)$ modification of Newton's constant, our best measurements of G come from torsion balance tests of Newton's

Law, and because we do not have very good measurements of G from other tests of GR, we can simply roll the $1/(1-\epsilon^2)$ modification into a redefinition of G. Also note that there it would be challenging to measure the Yukawa-like e^{-r/r_0} modification because it is strongest at length scales $r < 1/gM$, but this corresponds to $k > gM$ which is generically outside the range of validity of our effective theory.

The strongest constraints on this theory seem to come from looking at couplings to the Standard Model that are generated through graviton loops. Given a dimension four standard model operator $O^{\mu\nu}$ there is no symmetry forbidding us from writing down the interaction

$$\mathcal{L}_{\text{int}} = \frac{M^2}{M_{\text{Pl}}^4} \mathcal{O}^{\mu\nu} D_\mu \phi D_\nu \phi, \tag{43}$$

where the factor $1/M_{\text{Pl}}^4$ is the naive suppression factor for a dimension 8 operator in an effective field theory with a Planck scale cutoff, and the factor of M^2 accounts for the fact that in our conventions, ϕ has dimension zero. When $D_\mu \phi$ takes its vev, we are left with the operator

$$\mathcal{L}_{\text{int}} = \frac{M^4}{M_{\text{Pl}}^4} \mathcal{O}^{00}. \tag{44}$$

One interesting candidate for $O^{\mu\nu}$ is the stress-energy tensor $T^{\mu\nu}$. Adding a term T^{00} modifies the maximum attainable velocity for various particles, and it can be shown that this places a bound $M < 10^w$ GeV. In the case of the original ghost condensate, this bound was trivial compared to the constraints imposed by gravity, but in the gauged ghost condensate, this bound is (apparently) the only guaranteed bound on the scale M if we are free to push g to be much larger than $M/M_{\text{Pl}} \sim 10^r$.

Of course, we could certainly imagine other couplings to $D_\mu \phi$. For example, if we relax the assumption of a $\phi \to -\phi$ symmetry, then we could have a coupling to, say, the electromagnetic current:

$$\mathcal{L}_{\text{int}} = \frac{M}{F} J^\mu D_\mu \phi, \tag{45}$$

where F is some unknown mass scale. In the case of the ungauged ghost condensate, we could phase away this interaction via a field redefinition of the fermions Ψ ($J^\mu = \bar{\Psi}\gamma^\mu \Psi$), but now there is no way to phase away the piece of the coupling

$$\mathcal{L}_{\text{int}} = \frac{M}{F} J^\mu A_\mu. \tag{46}$$

Though this looks like the standard electromagnetic interaction, we have to remember that even in the absence of gravity, A_0 mixes with the Goldstone boson. This modifies the $\langle A_0 A_0 \rangle$ propagator from $1/k^2$ to $1/(k^2 + g^2 M^2)$, but it does not substantially modify the $\langle A_i A_i \rangle$ propagator. This yields an additional Yukawa-like interaction between electric charges but an additional undamped interaction between electric currents. Therefore, at distances larger than $1/gM$, one would observe that the effective α_{EM} for charge-charge interactions would differ from the effective α_{EM} for current-current interactions by $(M/F)^2$. Given the amazingly precise measurements of α_{EM} in a wide range of experiments, we expect stringent bounds on this direct coupling.

7 Prospects

We have seen that by charging our ghost scalar under a $U(1)$ gauge symmetry, we have given ourselves new freedom to adjust the modification of gravity due to ghost condensation. For values of the gauge coupling larger than $M/M_{\rm Pl}$, we not only remove the Jeans instability, but we also end up with modification of Newton's Law that might as well not be a modification at all. Whether this is an improvement or not depends on whether we think signals of Lorentz-violation will appear in gravitational physics.

At the very least, we have shown that it is possible to have spontaneous Lorentz-violations at very high energies. It should come as no surprise to anyone that we are only able to do this by spontaneously breaking Lorentz-invariance in a hidden sector. Still, we know that gravity couples universally, so it is a bit bizarre that gravity is only softly modified if the gauge coupling is large enough. As mentioned already, qualitatively we can think of the Goldstone boson as being "less eaten" by the graviton as we increase the gauge coupling. Alternatively, we can think in terms of adjusting the "mixing angle" of the unbroken $U(1)$ gauge symmetry such that the unbroken $U(1)$ is mostly time diffeomorphisms.

To really understand whether gauged ghost condensation is truly a viable alternative to ungauged ghost condensation, we need to more thoroughly understand all the experimental limits. While it appears that we are safe even with a symmetry breaking scale as high as $M = 10^w$ GeV, there may be other effects that we aren't considering. Also, it is essential that we understand whether the gauged ghost condensate looks at all like cold dark matter and whether there might be an inflation scenario based on a gauged ghost field.

We could also consider more complicated symmetry breaking patterns that still preserve $SO(3)$ invariance. For example, we could consider space diffeomorphisms and an $SU(2)$ gauge group being spontaneously broken to a diagonal $SU(2)$, and this diagonal $SU(2)$ might even confine. While it is not clear whether such a theory is well-behaved, it may be worthwhile to catalog all possible Lorentz-breaking patterns to understand the range of consistent infrared modifications of gravity.

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