Universal Dynamics of Spontaneous Lorentz-Violation and a New Spin-Dependent Inverse-Square Law Force

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hep-ph/0407034

Question: What if the vacuum state of the universe is not Lorentz-invariant?

Answer (Without Gravity): Write down an effective theory with non-zero vevs for various tensor operators.

$$\mathcal{L}_{\mathrm{CPT-odd}} = b_0 \bar{\Psi} \gamma^0 \gamma^5 \Psi$$

Colladay and Kostelecký: `hep-ph/9809521`

Goal: Measure the spurion b_0.

Experiment: Electrons with $v \sim 10^{-3}$:

$$b_0 < 10^{-25} \text{ GeV}$$

Heckel, et. al. EotWash, 1999.

Answer (With Gravity): *Forced* to introduce new propagating degree of freedom!

The Goldstone Boson of Spontaneous Lorentz-Violation

Key Point: For every spurion there is a new dynamical effect mediated by the Goldstone.

Complementary tests for Lorentz-violations!

Goldstone Boson

Lorentz-Violating Dispersion Relation:

$$\pi : \quad \omega^2 = \frac{k^4}{M^2}$$

Leading Coupling to SM:

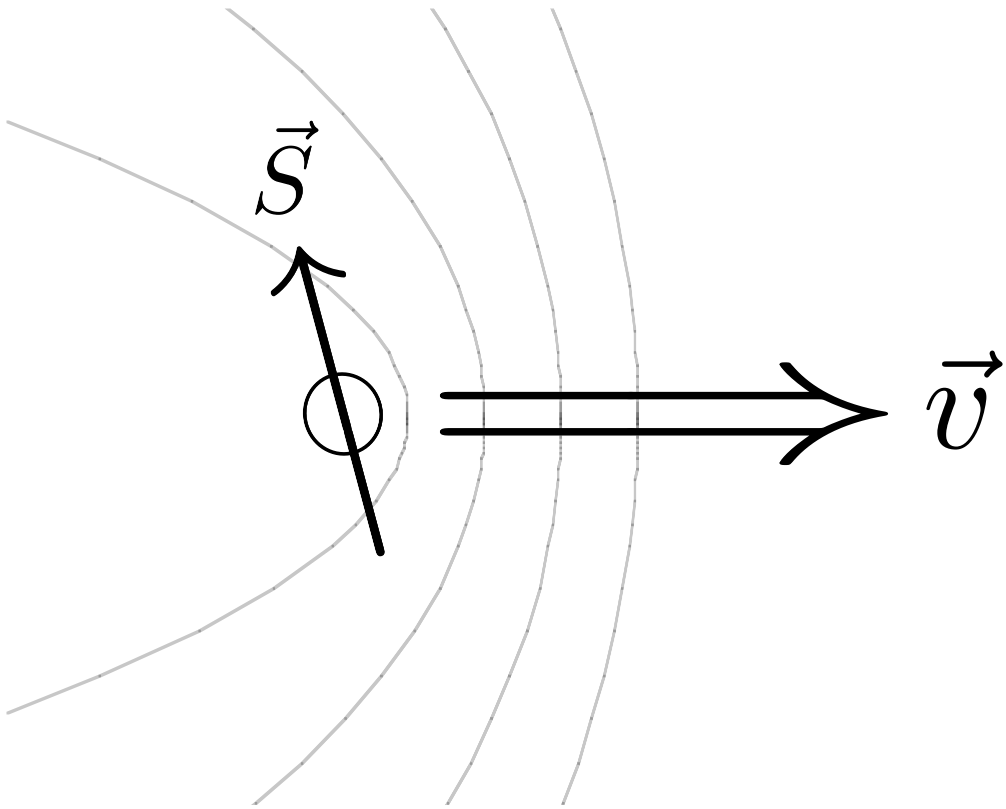
$$\mathcal{L}_{\rm int} = \frac{1}{F}\left(M^2 \bar{\Psi}\gamma^0\gamma^5\Psi - \bar{\Psi}\vec{\gamma}\gamma^5\Psi \cdot \vec{\nabla}\pi\right)$$

Two New Dynamical Effects!

“Ether” Cherenkov Radation

Long-Range Spin-Dependent Force

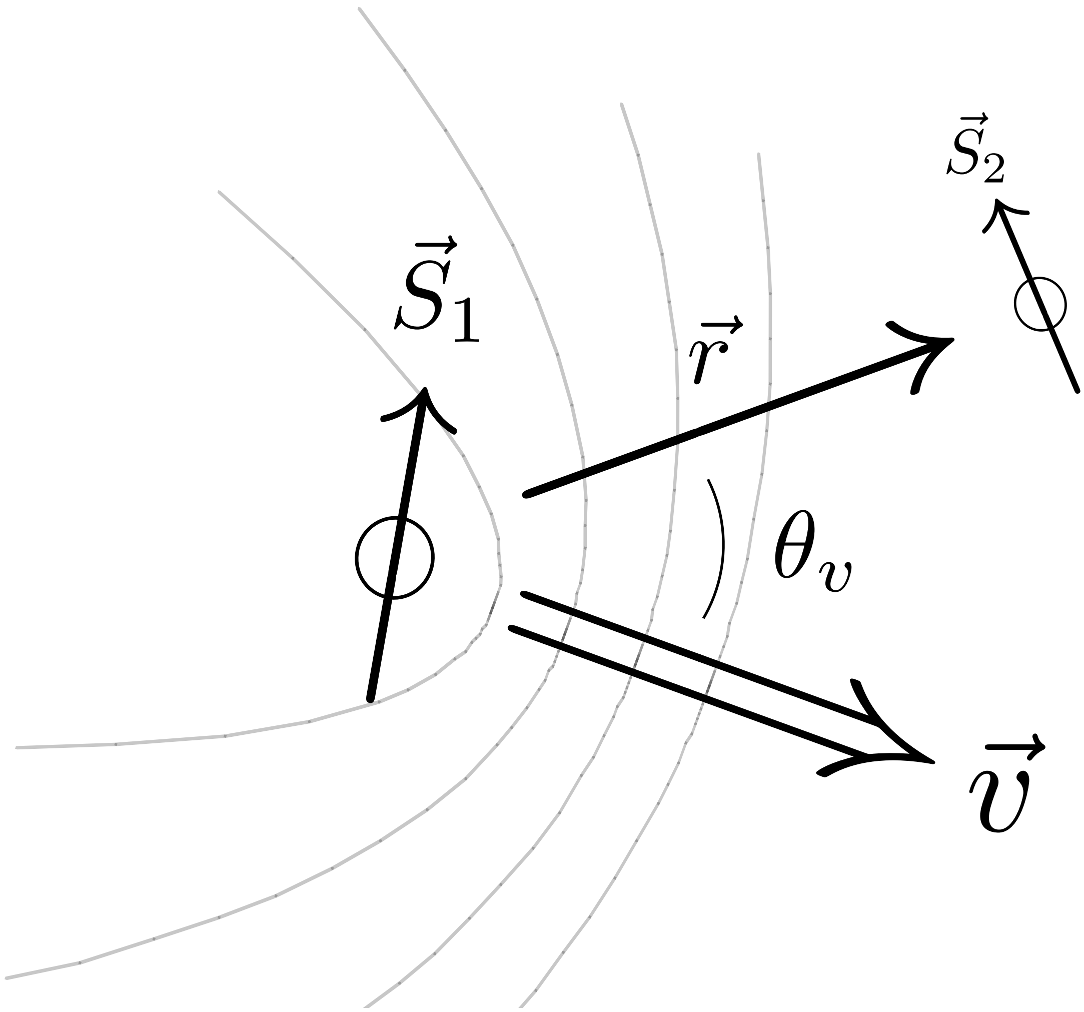
Ether Cherenkov Radiation



Velocity of Goldstone excitations is $v \sim k/M$. Always a k mode such moving particle is traveling faster than it. Goldstone shockwave!

Note: Sometimes, effective theory breaks down before experimental bound.

Long-Range Spin-Spin Force



Massless bosons $\Longrightarrow$ long-range forces.

Normally: $1/r^3$ spin-spin potential.

Goldstone: $1/r$ spin-spin potential!

Spurion bounds $\Longrightarrow$ some dynamical bounds

Why Does Gravity Matter?

Diffeomorphisms: $x^\mu \to x^\mu + \xi^\mu(x)$

$$g_{\mu\nu} \text{ (10 d.o.f.)} \implies \text{Graviton (2 modes)}$$

$$\Delta\mathcal{L} = J^0$$

No way to covariantize with $g_{\mu\nu}$!

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\begin{aligned} h_{\mu\nu} &\to h_{\mu\nu} - \partial_\mu\xi_\nu - \partial_\nu\xi_\mu + \ldots \\ J^0 &\to J^0 - J^\mu\partial_\mu\xi^0 - \xi^\mu\partial_\mu J^0 + \ldots \end{aligned}$$

Broken Diffeo: No conceptual problem. Just new polarizations for graviton.

Stückelberg Trick

Introduce field that shifts under ξ^0:

$$\pi \to \pi - \xi^0$$

Goldstone Boson of Time Diffeomorphism Breaking (and Lorentz-Violation)

$$\begin{aligned} h_{\mu\nu} &\to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu \\ J^0 &\to J^0 - J^\mu \partial_\mu \xi^0 - \xi^\mu \partial_\mu J^0 \end{aligned}$$

Covariant combination involving J^0 and π to leading order ($M_{\rm Pl} \to \infty$):

$$\mathcal{L}_{\rm int} = J^0 - J^\mu \partial_\mu \pi$$

Arkani-Hamed, Cheng, Luty, Mukohyama: `hep-th/0312099`

Standard Model Couplings

Equivalently: $\phi \to \phi + c$

$$\langle \partial_\mu \phi \rangle = M^2 \delta^0_\mu \qquad \phi = M^2 t - \pi$$

$$\mathcal{L}_{\rm int} = \frac{1}{F} J^\mu \partial_\mu \phi \implies \frac{M^2}{F} J^0 - \frac{1}{F} J^\mu \partial_\mu \pi$$

Leading coupling: $J^\mu = \bar{\Psi}\gamma^\mu\gamma^5\Psi$

Non-relativistic limit:

$$\mathcal{L}_{\rm int} = \frac{M^2}{F}\bar{\Psi}\gamma^0\gamma^5\Psi + \frac{1}{F}\vec{s}\cdot\vec{\nabla}\pi$$

New derivative coupling to spin density!

Experimental Bound: $M^2/F < 10^{-25}$ GeV

Kinetic Term for Goldstone

$$\begin{aligned} h_{00} &\rightarrow h_{00} - 2\partial_0 \xi_0 \\ h_{0i} &\rightarrow h_{0i} - \partial_0 \xi_i - \partial_i \xi_0 \\ h_{ij} &\rightarrow h_{ij} - \partial_i \xi_j - \partial_j \xi_i \\ \pi &\rightarrow \pi - \xi_0 \end{aligned}$$

Can we covariantize $\dot{\pi}^2$?

$$M^4 \left(\dot{\pi} - \frac{1}{2} h_{00} \right)^2$$

What about $(\nabla \pi)^2$?

$$\delta(\partial_i \pi - h_{0i}) \rightarrow -\partial_0 \xi_i$$

Kinetic Term for Goldstone

$$\begin{aligned} h_{00} &\rightarrow h_{00} - 2\partial_0 \xi_0 \\ h_{0i} &\rightarrow h_{0i} - \partial_0 \xi_i - \partial_i \xi_0 \\ h_{ij} &\rightarrow h_{ij} - \partial_i \xi_j - \partial_j \xi_i \\ \pi &\rightarrow \pi - \xi_0 \end{aligned}$$

For π to propagate, we must go to $(\nabla\pi)^4$!

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j} + \partial_i \partial_j \pi\right)$$

$$\delta K_{ij} = 0$$

Spatial kinetic pieces for π:

$$K_{ij}^2, K_{ii}^2 \rightarrow (\nabla^2 \pi)^2$$

Kinetic Term for Goldstone

Even in $M_{\rm Pl} \to \infty$ limit, residual space diffeomorphisms force

$$\pi : \quad \omega^2 = \frac{k^4}{M^2}$$

Mixing with Gravity:

$$\omega^2 = -\frac{M^2}{2M_{\rm Pl}^2} k^2 + \frac{k^4}{M^2}$$

Bounds on M:

$$r_c \sim \frac{M_{\rm Pl}}{M^2} \qquad r_c > H_0^{-1} \Rightarrow M < 10^{-3} \text{ eV}$$

$$t_c \sim \frac{M_{\rm Pl}^2}{M^3} \qquad t_c > H_0^{-1} \Rightarrow M < 10 \text{ MeV}$$

Consistent Effective Theory

Just the Goldstone Boson ($M_{\rm Pl} \to 0$):

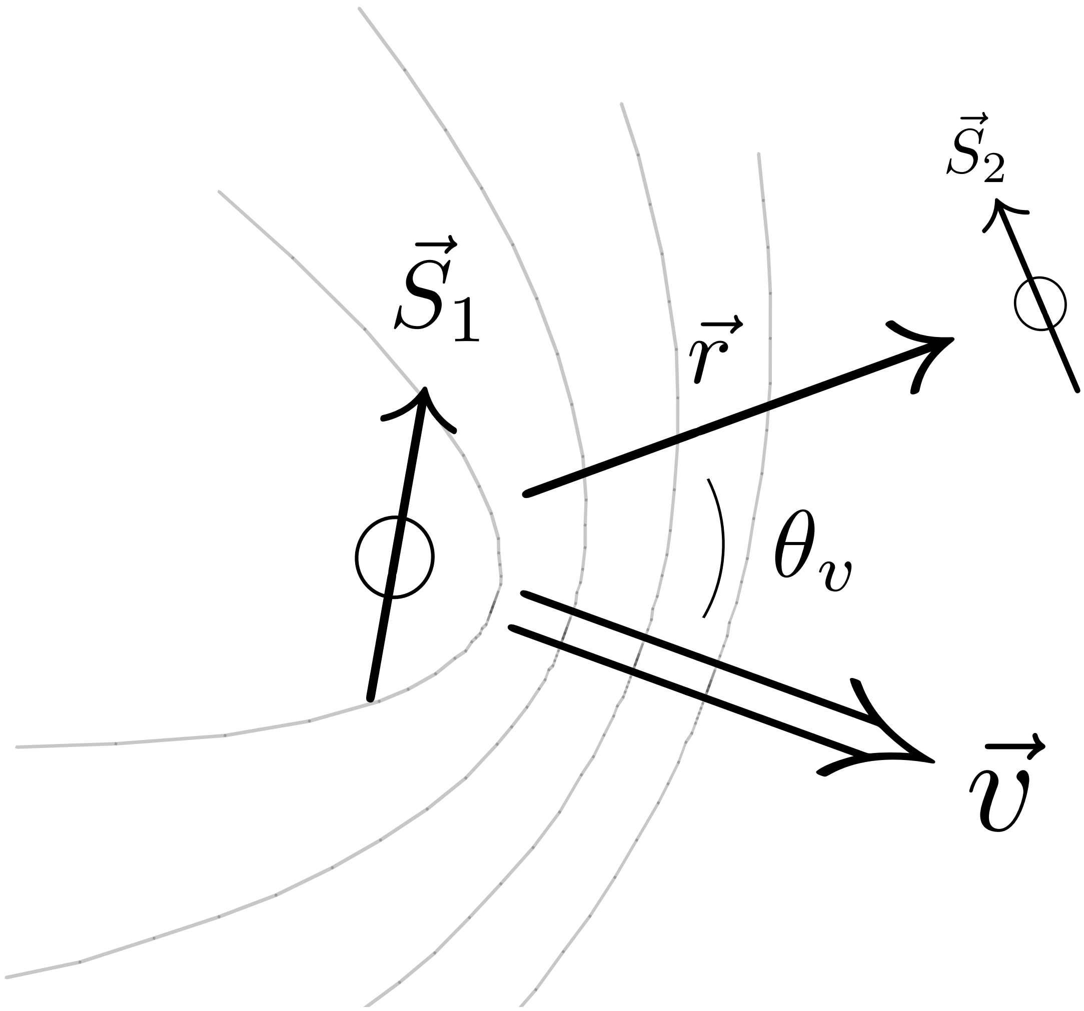
$$\mathcal{L} = \frac{1}{2}\dot{\pi}^2 - \frac{1}{2M^2}(\nabla^2\pi)^2 - \frac{\lambda}{2M^2}\dot{\pi}(\nabla\pi)^2 + \frac{\lambda}{8M^4}(\nabla\pi)^4 + \ldots$$

Coupled to Gravity (Unitary Gauge $\pi \equiv 0$):

$$\begin{aligned} \gamma_{ij} &= \text{spatial metric, constant time slice} \\ K_{ij} &= \text{extrinsic curvature from } \gamma_{ij} \\ R^{(4)} &= R^{(3)} + K_{ij}^2 - K^2 \end{aligned}$$

$$\mathcal{L} = \sqrt{g}R^{(4)} + \sqrt{\gamma}\left(M^4(g^{00}-1)^2 - \alpha_1 M^2 K^2 - \alpha_2 M^2 K_{ij}^2 + \ldots\right)$$

Long-Range Spin-Spin Force



Goldstone of L.V. mediates a $1/r$ potential!

Ingredients:

$$\frac{1}{F}\vec{s}\cdot\vec{\nabla}\pi \qquad \omega^2 = \frac{k^4}{M^2}$$

Review of Usual Goldstones

Goldstone φ with $\omega \sim k$ dispersion relation.

Typical pseudoscalar interaction:

$$\mathcal{L}_{\rm int} = \frac{1}{F} \vec{s} \cdot \vec{\nabla} \varphi$$

Non-relativistic ($\omega = 0$) potential:

$$V_\varphi(r) = \frac{-1}{F^2} (\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla}) \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} e^{i\vec{k}\cdot\vec{r}}$$

The potential is identical to magnetic dipoles:

$$V_\varphi(r) = \frac{1}{4\pi F^2} \frac{(\vec{S}_1 \cdot \vec{S}_2) - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})}{r^3}$$

Wilczek and Moody, 1984.

Potential with Goldstone of L.V.

π: $\omega \sim k^2/M$ dispersion relation.

Leading Standard Model coupling:

$$\mathcal{L}_{\rm int} = \frac{1}{F} \vec{s} \cdot \vec{\nabla} \pi$$

Non-relativistic ($\omega = 0$) potential:

$$V_\pi(r) = \frac{-1}{F^2} (\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla}) \int \frac{d^3k}{(2\pi)^3} \frac{M^2}{k^4} e^{i\vec{k}\cdot\vec{r}}$$

A new long-range inverse-square law force!

$$V_\pi(r) = \frac{M^2}{8\pi F^2} \frac{(\vec{S}_1 \cdot \vec{S}_2) - (\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})}{r}$$

Three Considerations

- **Time Effects**
  What does non-relativistic limit mean? $\omega \ll k^2/M$ implies $t \gg Mr^2$. How long do we have to wait to see potential?

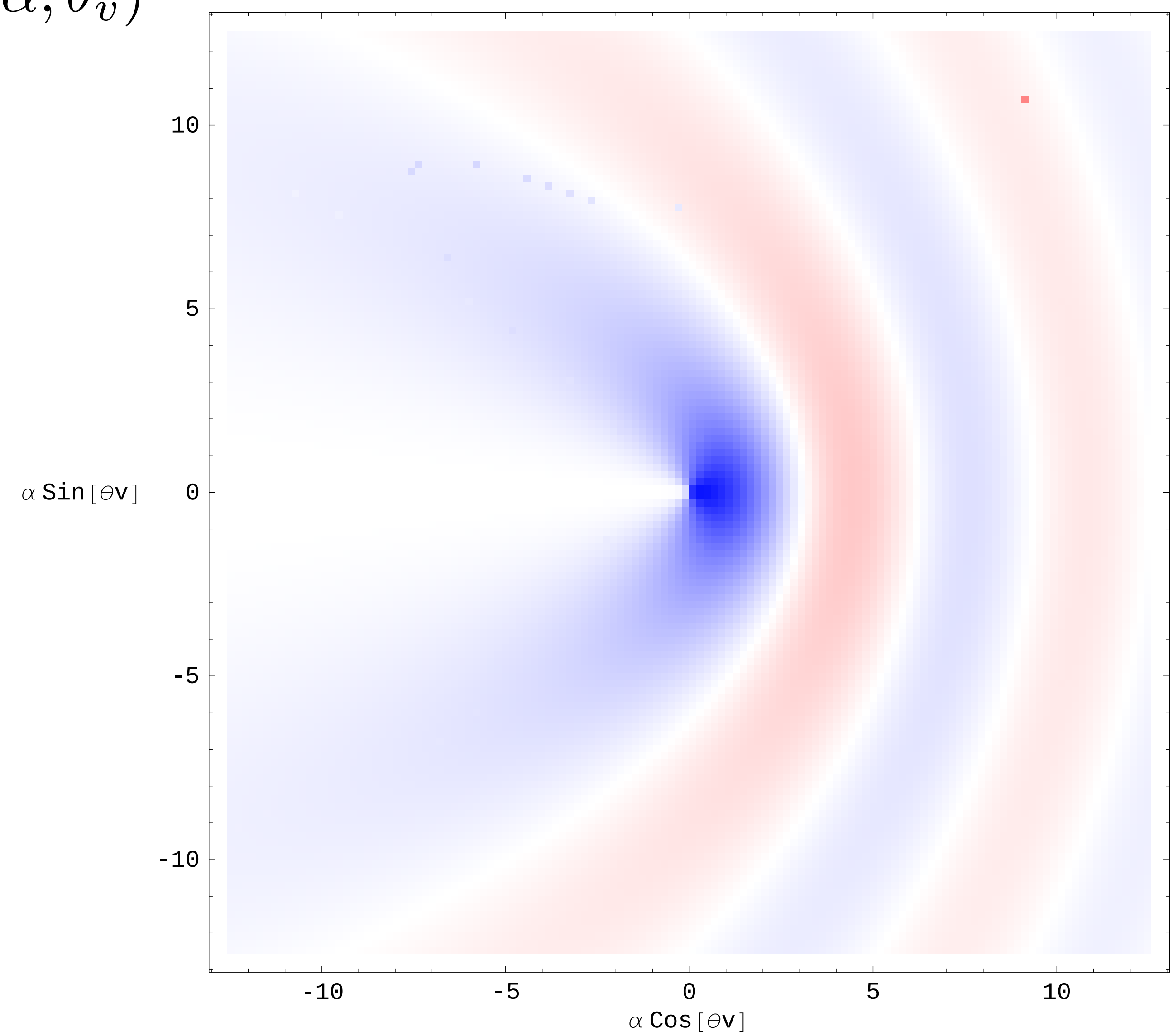
- **Size Effects**
  Does $\vec{s}$ coherently source π? Is there $\gamma = MRv$ suppression?

- **Velocity Effects**
  What does the potential look like if the source is moving with respect to the ether rest frame? Can the π field "keep up" with the moving source?

$$\alpha = Mrv \qquad \cos\theta_v = \hat{r}\cdot\hat{v}$$

$$V(r) = \frac{M^2}{8\pi F^2}\left(A(\alpha,\theta_v)\frac{\vec{S}_1\cdot\vec{S}_2 - (\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})}{r} + B(\alpha,\theta_v)\frac{(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})}{r}\right)$$

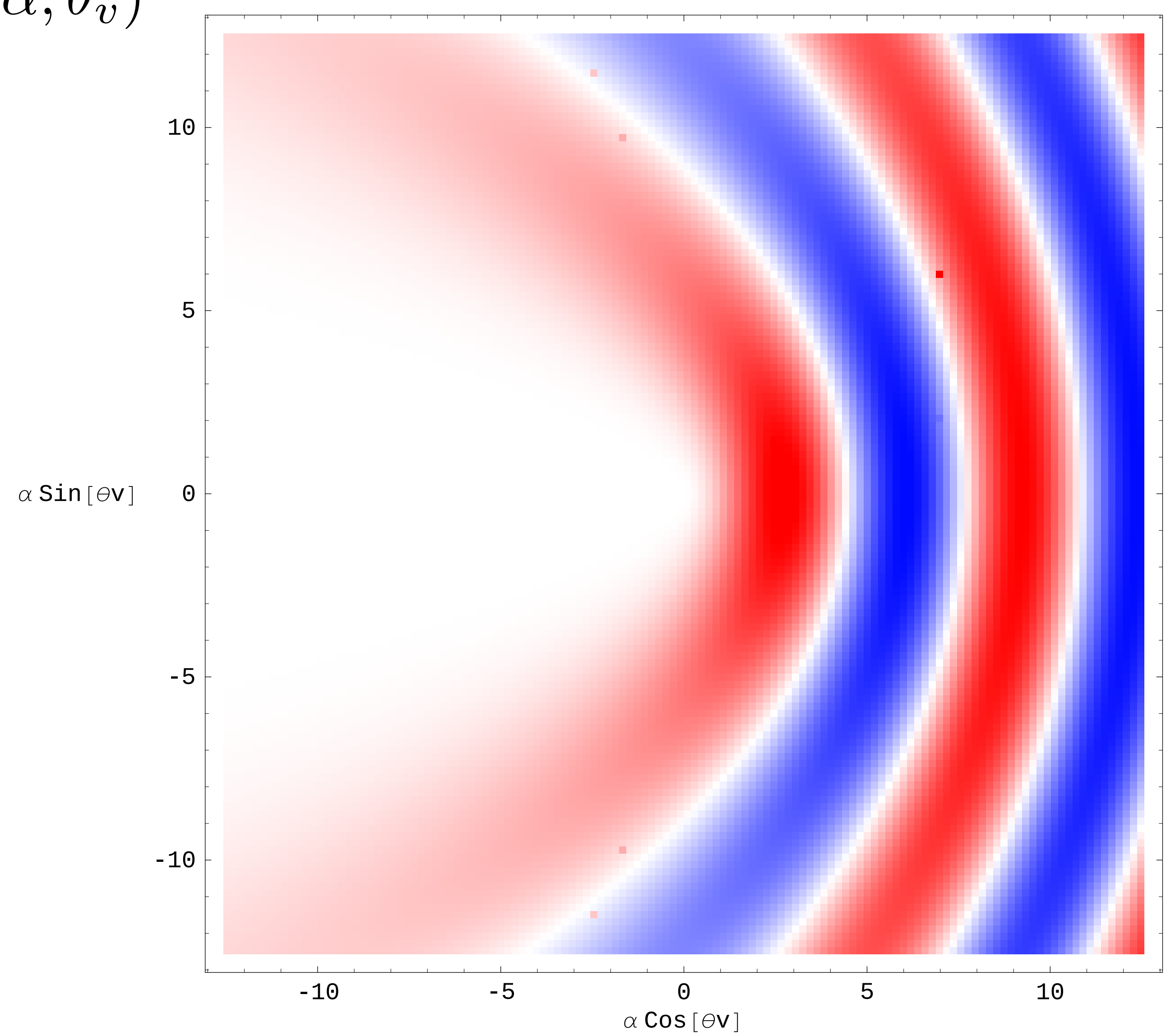
$A(\alpha,\theta_v)$



$$\alpha = Mrv \qquad \cos\theta_v = \hat{r}\cdot\hat{v}$$

$$V(r) = \frac{M^2}{8\pi F^2}\left(A(\alpha,\theta_v)\frac{\vec{S}_1\cdot\vec{S}_2 - (\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})}{r} + B(\alpha,\theta_v)\frac{(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})}{r}\right)$$

$B(\alpha,\theta_v)$



Experimental Possibilities

Spin force competes with gravity if:

$$\frac{M}{F} \sim \frac{1\ \text{GeV}}{M_{\text{Pl}}} \sim 10^{-19}$$

(One aligned spin per nucleon.) Same as $1/r^3$ search bound! Ritter, et. al. 1990

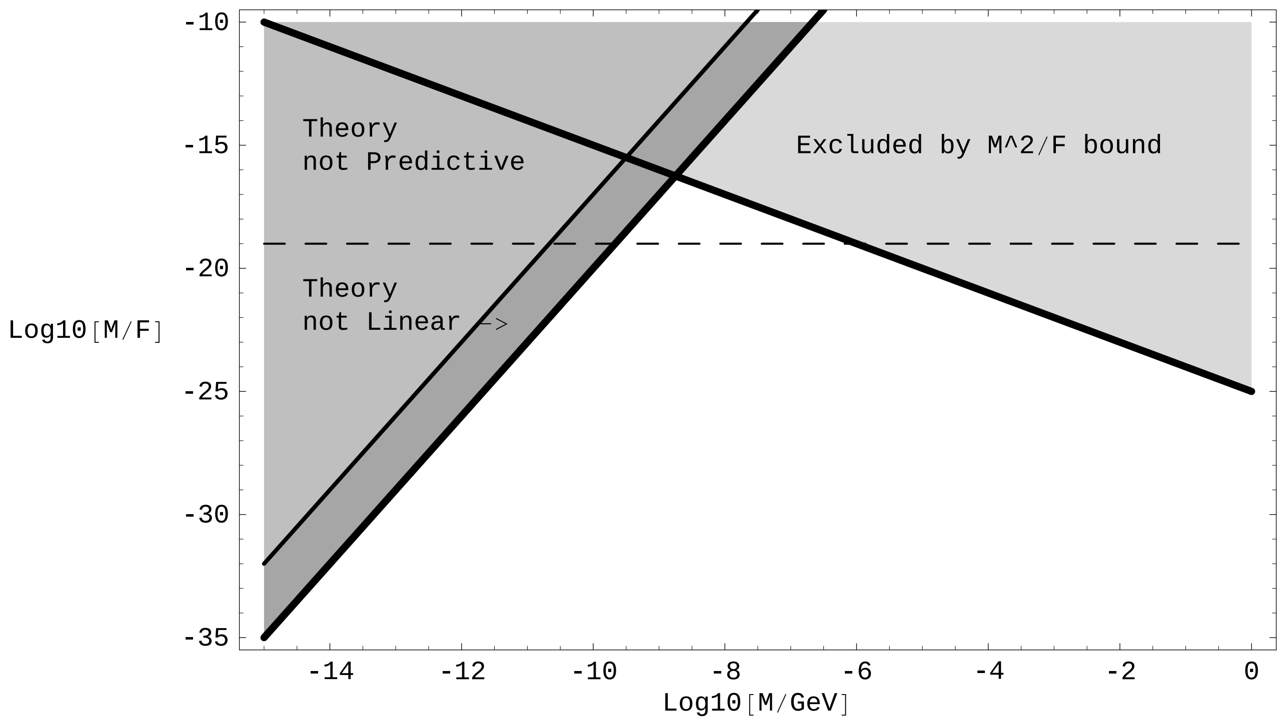
Unique angular dependence! Preferred direction in space! Daily and yearly effects!

Consistent, universal, dynamical Lorentz-violating theory with sharp predictions!

(The presence of the ghostone boson would be as shocking to the physics community today as the absence of the luminous ether was in 1887?)

Alnico Magnet Source

Large sources can push theory out of linear/predicitve regimes. Alnico magnet with $MRv = \gamma \sim 1$.



$M \sim 1$ eV and $v \sim 10^{-3}$ gives gravitational strength interaction with $R \sim .1 - 1$ mm.

Gauged Ghost Condensation

Time Diffeo $\times U(1) \to U(1)$:

$$\begin{aligned} h_{\mu\nu} &\to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu \\ A_\mu &\to A_\mu - \partial_\mu \alpha \\ \pi &\to \pi - \xi^0 - \alpha/M \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{EH} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + M^4 \left(\dot{\pi} - \frac{A_0}{M} - \frac{h_{00}}{2} \right)^2$$

$$- \left(\nabla^2 \pi - \frac{\vec{\nabla} \cdot \vec{A}}{M} - \partial h \right)^2 + J^0 + \vec{J} \cdot \left(\vec{\nabla} \pi - \vec{A} \right)$$

Vev for Charged Scalar:

$$\phi \to \phi - \alpha \qquad A_\mu \to A_\mu - \partial_\mu \alpha$$

$$\langle D_\mu \phi \rangle = \langle \partial_\mu \phi - A_\mu \rangle \neq 0$$

Gauged Ghost Condensation

Polarizations:

$$\text{2 modes (“}A_\mu\text{”):} \qquad \omega^2 = k^2$$

$$\text{1 mode (“}\pi\text{”):} \qquad \omega^2 = g^2 k^2 + \frac{k^4}{M^2}$$

Ether Cherenkov Radiation for $v > g$.

Spin-Spin Potential ($v \ll g \ll 1$):

$$V(r) = \frac{1}{8\pi r}\left(\vec{S}_1 \cdot \vec{S}_2 - (\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})\right)$$

$$+ \frac{g^2}{8\pi r}\left(\vec{S}_1 \cdot \vec{S}_2 + (\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})\right)$$

Cheng, Luty, Mukohyama, JKT: to appear

Gauged Ghost Condensation

Mixing with Gravity:

$$\omega^2 = (g^2 - g_c^2)k^2 + \frac{k^4}{M^2} \qquad g_c = \frac{M}{\sqrt{2}M_{\rm Pl}}$$

Newton's Law ($v = 0$, $\epsilon = g_c/g$):

$$V(r) = \frac{Gm_1m_2}{r}\frac{1}{1-\epsilon^2}\left(1 + \epsilon^2 e^{-Mr\sqrt{g^2-g_c^2}}\right)$$

Just a redefinition of G!

Newton's Law ($g \gg g_c$, $v \ll g$):

$$V(r) = \frac{G'm_1m_2}{r}\left(1 + \frac{g_c^2 v^2}{2g^4}\sin^2\theta_v\right)$$

Orbital Torque: $M < g^2 10^{18}$ GeV

Review Article, Will: gr-qc/0103036

New Ingredient in the Search for Lorentz-Violations

The Goldstone Boson of Spontaneous Lorentz-Violation

Must exist in a theory with gravity!

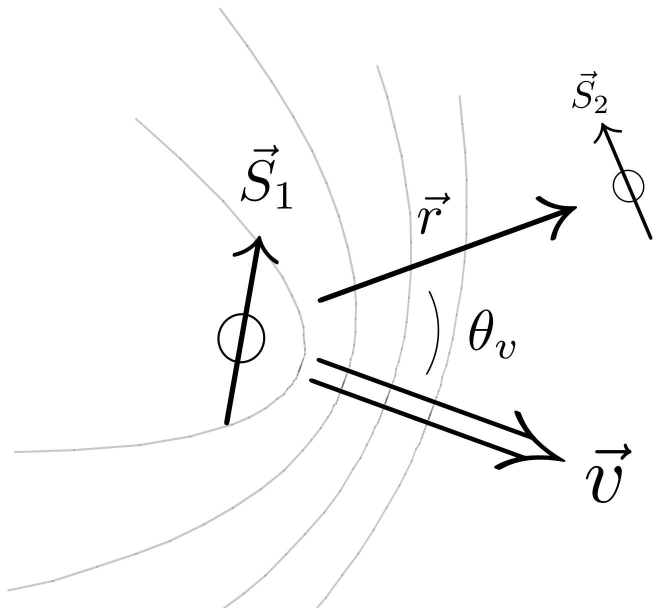
Kinetic terms have universal structure!

$\omega \sim k^2$: New dynamical effects!

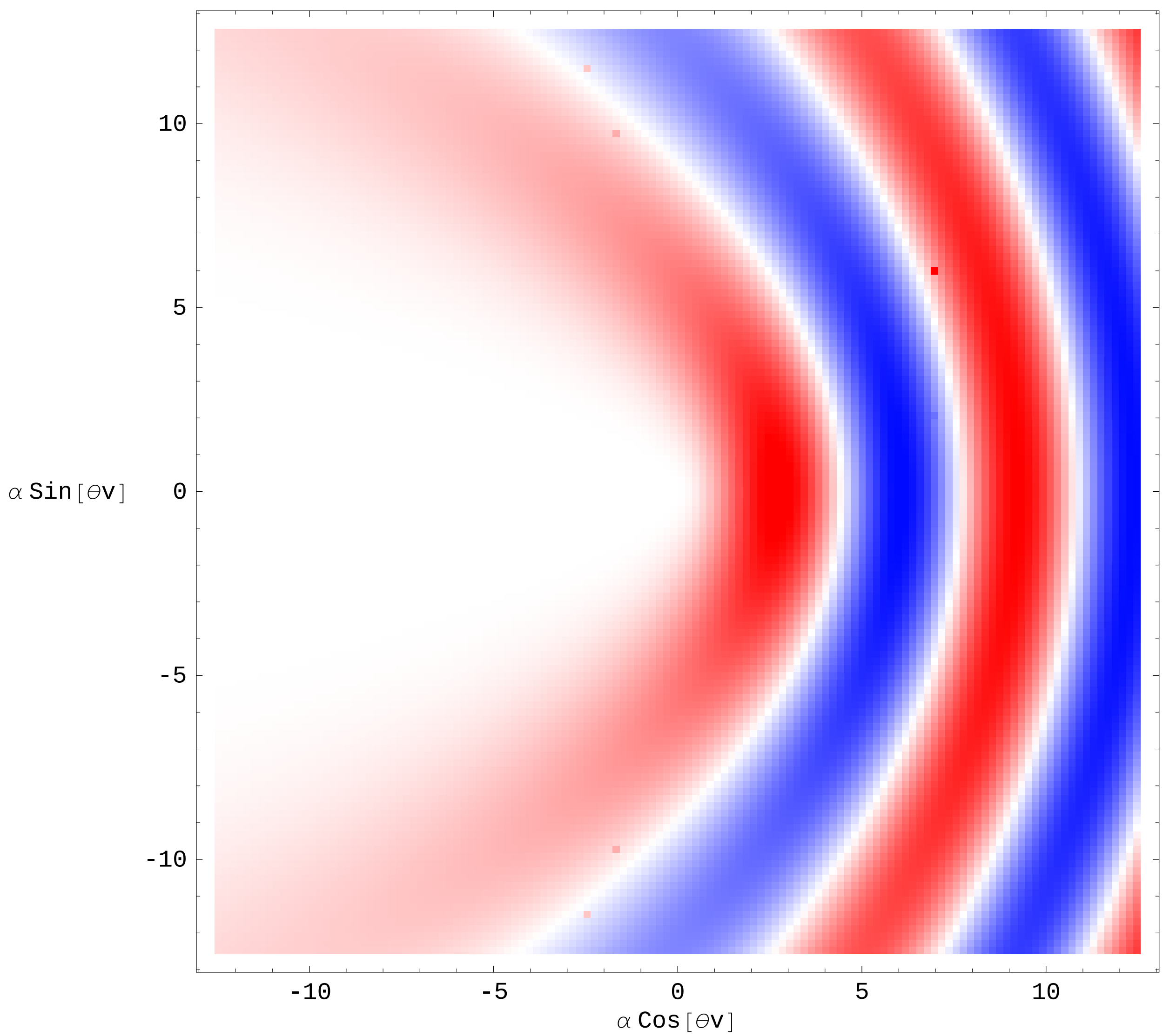
Long-Range Spin-Dependent Potential

1/r! Angular dependence!

Complements direct searches for Lorentz-violating operators.



$\vec{S}_2$
$\vec{S}_1$
$\vec{r}$
θ_v
$\vec{v}$



10
5
0
-5
-10
α Sin[θv]
-10
-5
0
5
10
α Cos[θv]