Spontaneous Time Diffeomorphism Breaking and Ghost Condensation

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1 Ghost Condensation

The theory of ghost condensation [1] offers us insight into a new consistent IR modification of gravity. It can also be used as an alternate model to slow-roll inflation [2], and in subsequent talks I will explore the connection between ghost condensation and Lorentz violating effects in the Standard Model.

But ghost condensation is certainly not the first IR modification of gravity that we have seen; a classic example is Einstein gravity with a Fierz-Pauli mass term. Experimental tests have ruled out massive gravity (or at least put tight constraints on its UV completion), but you may wonder whether we could follow the logic of the massive gravity construction to create a slightly different theory with greater experimental viability.

In essence, massive gravity is just a low energy effective field theory coming from spontaneous diffeomorphism breaking. The Goldstone bosons from the broken symmetries are eaten (in unitary gauge) to form the "longitudinal" modes of the now massive graviton. There is a slight twist, because diffeomorphism in 4 dimensions are parameterized by 4 degrees of freedom, whereas there are only 3 longitudinal modes for a $4D$ massive graviton. So we have to choose the mass term carefully to make sure there are no pathological kinetic terms from the extra Goldstone mode.

In this language, ghost condensation is equivalent to a low energy effective field theory coming from spontaneous time diffeomorphism breaking. There is only one Goldstone mode, and it is eaten (in unitary gauge) to form the "longitudinal" mode of the ghost condensed graviton. In this way, it is quite analogous to Fierz-Pauli massive gravity. Of course, both theories suffer from the problem that neither has a well-defined UV completion. In effect, we have no well-defined Higgs-like mechanism to generate the symmetry breaking (though in the case of ghost condensation, one could make the argument that we are seeing a gravitational Higgs effect). Regardless, at energies near and below the symmetry breaking scale, we have a nice, well-defined effective field theory.

In this paper, I will present the construction of the ghost condensate Lagrangian in the language of spontaneous symmetry breaking. Starting from the example of a massive $U(1)$ gauge boson, we will see how the idea of Goldstone bosons carries over from gauge symmetry breaking to diffeomorphism breaking. Our goal is to see how the Goldstone boson corresponding to broken time diffeomorphisms could couple to Standard Model fields, generating possibly observable Lorentz violating effects in SM sector.

2 A Review of Spontaneous Symmetry Breaking

We are all familiar with the example of a spontaneously broken $U(1)$ gauge symmetry. We have seen many times that the Goldstone boson corresponding to the broken symmetry is eaten to form the longitudinal mode of the now massive gauge boson. Above the symmetry breaking scale v we have the gauge symmetry

$$A_\mu \to A_\mu + \partial_\mu \alpha. \tag{1}$$

Below the scale v, this symmetry is spontaneously broken by whatever mechanism, but we can restore it formally by performing the broken symmetry and promoting it to a field ϕ. Our non-linear $U(1)$ gauge symmetry is

$$A_\mu \to A_\mu + \partial_\mu \alpha, \qquad \phi \to \phi - \alpha. \tag{2}$$

The relevant, Lorentz invariant, and gauge invariant interactions we can form are

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + v^2 (A_\mu + \partial_\mu \phi)^2, \tag{3}$$

where dimension-full couplings are just set by the symmetry breaking scale v. In unitary gauge, we set $\phi = 0$, and going to canonical normalization, we are left with the familiar Lagrangian of a massive $U(1)$ gauge boson:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + m^2 A_\mu A^\mu, \tag{4}$$

where $m = gv$ is the mass of the gauge field. If we want to couple matter to our gauge boson, we need only identify the charge of our particle under the $U(1)$ symmetry and form all gauge invariant interactions, being mindful to include couplings to the ϕ field.

Note that we can look at the limit as $g \to 0$ holding v fixed. The A_μ field decouples from the ϕ field, and we are left with (in canonical normalization):

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2. \tag{5}$$

This is just the standard Lagrangian for a massless scalar field, which tells us that our theory is well behaved in the decoupling limit.

What about the case of a non-Abelian gauge symmetry G being spontaneously broken to the subgroup H? Let T_a be the group generators of G. Above the symmetry breaking scale v, the gauge symmetry is

$$A_\mu^a T_a \to e^{i\alpha^a T_a} \left(A_\mu^a T_a + i\partial_\mu \right) e^{-i\alpha^a T_a} \tag{6}$$

Below the scale v, this symmetry is spontaneously broken, but we can easily restore it. Let T_A be the broken symmetry directions. The new, non-linear realization of the gauge symmetry G is

$$A_\mu^a T_a \to e^{i\alpha^a T_a} \left(A_\mu^a T_a + i\partial_\mu \right) e^{-i\alpha^a T_a}, \qquad U \to e^{i\alpha^A T_A} U e^{-i\alpha^A T_A}. \tag{7}$$

The relevant, Lorentz invariant, and gauge invariant interactions we can form are now

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) + v^2\,\mathrm{tr}(D_\mu U D^\mu U^\dagger), \tag{8}$$

where $F_{\mu\nu}$ is the standard Faraday tensor of non-Abelian gauge symmetries, and $D_\mu = \partial_\mu - iA^A_\mu T_A$ is the gauge covariant derivative in the broken directions only. In unitary gauge, we set $U = 1$ and arrive at the Lagrangian of a gauge boson with $\dim(G)-\dim(H)$ massive modes corresponding to the $\dim(G)-\dim(H)$ Goldstone bosons that were eaten:

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) + m^2 A^A_\mu A^\mu_A, \tag{9}$$

where again $m = gv$. The physics in terms of the U field is much easier to understand than the physics in unitary gauge because the Lagrangian for U is just a (gauged) non-linear sigma model. If we go to the decoupling limit, then we find:

$$\mathcal{L} = v^2\,\mathrm{tr}(\partial_\mu U \partial^\mu U^\dagger) \tag{10}$$

which is just a regular non-linear sigma model for the broken symmetry directions. If we expand $U = e^{i\phi^A T_A}$, then the low energy physics in the decoupling limit is described by the interactions of ϕ^A.

The advantage of introducing ϕ or U is threefold. First, it isolates scattering amplitudes that involve the longitudinal mode of the massive gauge boson. This is especially important in theories like massive non-Abelian gauge theories where the longitudinal modes become strongly coupled at the scale $4\pi v$, and where naive Feynman diagram analysis gives the wrong answer.

Second, Goldstone bosons enable us to understand the structure of higher order terms in the Lagrangian. Naively, one might think that if gauge symmetry is broken at a scale v, then terms like $(\partial_\mu A^\mu)^2$ could appear in the low energy Langrangian. While this is true, the coefficient of this term is not arbitrary because it really comes paired with a $(\Box\phi)^2$ term and is therefore suppressed by two powers of the cutoff $\Lambda = 4\pi v$. Thus, the coefficient of the $(\partial_\mu A^\mu)^2$ term is actually $(v/\Lambda)^2$, so the term $(\partial_\mu A^\mu)^2$ is actually suppressed by a factor of $1/16\pi^2$.

Third, Goldstone bosons sometimes make it easier to envision a possible UV completion of the model (such as the Higgs mechanism). Note, however, that we need not have a UV completion in hand to justify the low-energy Lagrangian in equation (3). It is a *universal* form that depends only on the existence of — and not the specific mechanism of — spontaneous symmetry breaking.

3 Spontaneous Diffeomorphism Breaking and Massive Gravity

The key to understanding massive gravity in the language of spontaneous symmetry breaking is to regard gravity as merely another gauge theory. If we linearize gravity around a flat background, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, then diffeomorphisms are just the gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu. \tag{11}$$

To preserve diffeomorphism invariance, the quadratic piece of the Lagrangian must take the form

$$\mathcal{L} = \frac{1}{2}(\partial_\omega h_{\mu\nu})^2 + (\partial^\mu h)(\partial^\nu h_{\mu\nu}) - \frac{1}{2}(\partial_\mu h)^2 - (\partial^\nu h_{\mu\nu})^2, \tag{12}$$

where we raise and lower indices with respect to the flat metric $\eta^{\mu\nu}$ and $h = h^\mu_\mu$. This is just the linearization of the Einstein action,

$$\mathcal{L} = \sqrt{g}R. \tag{13}$$

(If we want to be really careful, we can track all factors of $\lambda = \sqrt{8\pi G}$ in order that $h_{\mu\nu}$ has mass dimension 1. For a rough idea of how this works, see [4].)

What if we spontaneously break diffeomorphism invariance? Comparing to the case of the massive gauge boson, we expect that the Goldstone bosons will form the "longitudinal" modes of the spin-2 boson. We can formally restore diffeomorphism invariance by introducing a Goldstone vector A_μ. (This is in keeping with the notation of [3]).

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \qquad A_\mu \to A_\mu - \xi_\mu. \tag{14}$$

We still have all of the terms in the massless Lagrangian from equation (12), but we can also form the following relevant, Lorentz invariant, and gauge invariant mass terms:

$$\Delta\mathcal{L} = \alpha v^2 (h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu)^2 + \beta v^2 (h + 2\partial^\mu A_\mu)^2, \tag{15}$$

where v is the symmetry breaking scale, and α and β are coefficients we are free to choose.

In the case of broken gauge symmetry, the choice of any $\mathcal{O}(1)$ coefficients did not affect our theory, but now, we want to make sure that the kinetic terms for A_μ have the right form in order for our theory not to have ghosts or tachyons in the decoupling limit. In essence, the fact that there are 4 Goldstone modes but only 3 allowed longitudinal modes means that we should expect to do some fine-tuning to arrive at a healthy theory.

If we set $h_{\mu\nu} = 0$ (equivalently, if we send $M_{pl} \to \infty$), we can isolate the A_μ kinetic terms.

$$\Delta\mathcal{L} = \alpha v^2 (\partial_\mu A_\nu + \partial_\nu A_\mu)^2 + \beta v^2 (2\partial^\mu A_\mu)^2 = \alpha v^2 (F_{\mu\nu})^2 + (2\alpha + 2\beta) v^2 (\partial^\mu A_\mu)^2 \tag{16}$$

The first term looks like a perfectly reasonable kinetic term for a massless spin-1 field, albeit with a non-standard normalization. The second term breaks $U(1)$ gauge invariance for A_μ, but we can formally restore it (and isolate the spin-0 longitudinal mode of the massive graviton) by performing the broken gauge transformation and promoting it to a field ϕ:

$$(\partial^\mu A_\mu)^2 \to (\partial^\mu A_\mu + \Box\phi)^2. \tag{17}$$

So the kinetic term for ϕ is

$$\Delta\mathcal{L} = (2\alpha + 2\beta) v^2 (\Box\phi)^2. \tag{18}$$

This gives a dispersion relation involving p^4 instead of p^2, which implies that the ϕ external states could be ghosts or tachyons. In order to eliminate this pathologinal kinetic term, we should choose $\alpha = -\beta$. Going to unitary gauge, equation (15) is precisely the Fierz-Pauli mass term,

$$\Delta\mathcal{L} = \alpha v^2 \left((h_{\mu\nu})^2 - h^2 \right). \tag{19}$$

In this way, we have recreated the quadratic part of the massive gravity Lagrangian.

There is much more to the story of massive gravity. In particular, in this flat space case, a kinetic term for ϕ is generated through mixing with $h_{\mu\nu}$. Also, choosing different vacua to expand $g_{\mu\nu}$ around give different results in the decoupling limit. And looking at the interactions of the A_μ and ϕ fields tells us at what mass scale we expect a violation of unitarity. But the point is clear: by using a Goldstone boson analysis of the broken gauge symmetries, we can more easily understand the physics than in unitary gauge.

4 Spontaneous Time Diffeomorphism Breaking and Ghost Condensation

In the example of massive gravity we choose to break each of the diffeomorphism directions. Now, we can generate an effective field theory for the ghost condensate by choosing only to break time diffeomorphisms. Of course, there is no guarantee that our theory will make sense. In the case of massive gravity, only through kinetic mixing did the spin-0 component have a healthy kinetic term. But here we will find that the extra degree of freedom is well behaved, albeit with a Lorentz violating kinetic term leading to a $\omega^2 \propto k^4$ dispersion relation.

Once again, we introduce a Goldstone field that compensates for the broken time diffeomorphism symmetry. Our non-linear diffeomorphism is

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \qquad \pi \to \pi - \xi_0. \tag{20}$$

(To maintain my sanity, I chose a different sign convention than [1].) Clearly, the kinetic term of equation (12) is still invariant under this symmetry. But we can also form polynomials out of the following combinations:

$$H = h_{00} + 2\partial_0 \pi, \qquad K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j} - \partial_i \partial_j \pi\right). \tag{21}$$

Note that a term like $(h_{0i} + \partial_i \pi)$ is forbidden by the unbroken space diffeomorphism symmetry. So a possible mass terms for the $h_{\mu\nu}$ fields (and correspondingly, kinetic terms for π) are

$$\Delta\mathcal{L} = \frac{1}{8} M^4 H^2 - \tilde{M}_1^2 K^2 - \tilde{M}_2^2 (K_{ij})^2 - \frac{1}{2}\tilde{M}_3^3 H K, \tag{22}$$

where coefficients are slightly different than [1] and $K = K_{ii}$.

Going to the decoupling limit $M_{pl} \to \infty$, we can set $h_{\mu\nu} = 0$. Equation (22) becomes

$$\Delta\mathcal{L} = \frac{1}{2} M^4 \dot{\pi}^2 - \frac{1}{2}\tilde{M}^2 (\nabla^2 \pi)^2 - \frac{1}{2}\tilde{M}_3^3 \dot{\pi} \nabla^2 \pi, \tag{23}$$

where dots indicate time derivatives, ∇ is the spacial gradient and $\tilde{M}^2 = (\tilde{M}_1^2 + \tilde{M}_2^2)/2$. We could have eliminated the term proportional to $\tilde{M}_3$ by imposing time-reversal invariance, but in the next section we will see that there is an interesting effect if we do not make this assumption.

Right away, we see that there is a normal time-like kinetic piece for π but no $(\nabla\pi)^2$ spacial kinetic piece. Therefore, by breaking only time diffeomorphism invariance, we have broken Lorentz invariance in the decoupling limit. The dispersion relation for π is

$$w^2 = \frac{\tilde{M}^2}{M^4} k^4 - \frac{\tilde{M}_3^3}{M^4} w k^2. \tag{24}$$

We could continue in this manor to find the leading interactions of the π fields. Because of the funny π dispersion relation, the scaling dimension of the interaction is not the same as the mass dimension of the interaction, so we would have to do a bit of work to figure out which interaction is the most relevant. (It turns out to be $\dot{\pi}(\nabla\pi)^2$.) Leonardo will show us that this interaction leads to interesting cosmic microwave background signatures if the ghost field is the inflation. Taking a different route, we can look at the coupling of the π field to gravity itself to see directly an infrared modification of gravity. Devin will investigate whether such a modification could help explain dark matter or dark energy.

5 Coupling Matter to the Ghost Condensate

If ghost condensation can lead to Lorentz violating effects in the gravity sector, a natural question is whether these Lorentz violating effects can be communicated to the Standard Model sector. Because the SM couples to gravity, there will always be effects transmitted through graviton loops, but we expect these to be suppressed by powers of the Planck scale. What about direct coupling to standard model fields?

To understand this, we could go to the canonical ghost picture in which the ghost field ϕ achieves an expectation value $\langle\phi\rangle = M^2 t$. In that language, we see that the ghost can couple to any vector operator through

$$\mathcal{O}^\mu \partial_\mu \phi. \tag{25}$$

We want to find the same result, but in the language of spontaneous symmetry breaking.

First, we need to know how matter fields transform under unbroken diffeomorphisms. The linearized metric transforms as

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \dots, \tag{26}$$

where there are additional terms at $\mathcal{O}(\xi^2)$ and $\mathcal{O}(h\xi)$. To this order, a scalar transforms as

$$\phi \to \phi + \xi^\mu \partial_\mu \phi. \tag{27}$$

We want to define diffeomorphisms such that fields transform but derivatives do not. In other words, we want $\delta(\partial\phi) = \partial(\delta\phi)$:

$$\partial_\mu \phi \to \partial_\mu \phi + (\partial_\mu \xi^\nu)\partial_\nu \phi + \xi^\nu \partial_\nu(\partial_\mu \phi). \tag{28}$$

We expect that a vector should transform the same way as the divergence of a scalar, so

$$A_\mu \to A_\mu + (\partial_\mu \xi^\nu) A_\nu + \xi^\nu \partial_\nu A_\mu. \tag{29}$$

All these formulas can be checked by looking at the linearized forms of diffeomorphisms from GR. If we wanted to be really careful, we could figure out how a chiral spinor transformed under diffeomorphisms. This would involve understanding the spin connection, however, which is more sophisticated than we need for our purposes. It suffices to check that the combination $\psi\psi$ transforms as a scalar and the combination $\bar{\psi}\bar{\sigma}^\mu\psi$ transforms as a vector, which they do.

As a quick check that everything is working, we can show that the combination

$$\left(\eta^{\mu\nu} + \frac{1}{2} h \eta^{\mu\nu} - h^{\mu\nu}\right) \partial_\nu \phi \partial_\mu \phi \tag{30}$$

is invariant under diffeomorphisms. This is just the linearization of $\sqrt{g} g^{\mu\nu} \partial_\nu \phi \partial_\mu \phi$, which is the covariant kinetic term for a scalar.

If time diffeomorphisms are spontaneously broken, then all terms invariant under total diffeomorphisms will still survive, but we also expect new couplings to be possible. In particular, all coupling to H, K, and K_{ij} are possible as long as they are invariant under $SO(3)$ rotations (i.e. all i indices are contracted). We will look at couplings to H below, but couplings to K or K_{ij} are subleading because they involve more derivatives on the π fields.

What we really want is a direct coupling to vector couplings. In particular, we want to try to couple a vector quantity A_i to h_{0i} in a way that preserves space diffeomorphisms, and then perform the broken time diffeomorphism to figure out how A_i couples to π. Under space diffeomorphisms and summing over repeated indices:

$$\begin{aligned} h_{00} &\to h_{00}, \\ h_{0i} &\to h_{0i} + \partial_0 \xi_i, \\ h_{ii} &\to h_{ii} + 2\partial_i \xi_i, \\ A_0 &\to A_0 - (\partial_0 \xi_j) A_j - \xi_j \partial_j A_0, \\ A_i &\to A_i - (\partial_i \xi_j) A_j - \xi_j \partial_j A_i. \end{aligned} \tag{31}$$

(The extra minus signs are because we are using the mostly negative metric so $A_i B^i = -A_i B_i$.) To maintain $SO(3)$ rotation invariance, we have the following allowed coupling to leading order.

$$\mathcal{L}_{\text{int}} = \alpha h_{0i} A_i + \beta h_{ii} A_0 + \gamma A_0 + \delta h_{00} A_0. \tag{32}$$

Varying to leading order:

$$\delta(\mathcal{L}_{\text{int}}) = \alpha(\partial_0 \xi_i) A_i + \beta(2\partial_i \xi_i) A_0 + \gamma(-(\partial_0 \xi_j) A_j - \xi_j \partial_j A_0). \tag{33}$$

So to have an interaction piece invariant under the remaining space diffeomorphisms, we must choose the coefficients $\alpha = -2\beta = \gamma$. The value of δ (*i.e.* the coupling to H) is unfixed.

$$\mathcal{L}_{\text{int}} = \alpha \left(h_{0i} A_i - \frac{1}{2} h_{ii} A_0 + A_0 \right) + \delta h_{00} A_0. \tag{34}$$

Under the broken time diffeomorphism, these quantities transform as

$$\begin{aligned} h_{00} &\to h_{00} + 2\partial_0 \xi_0, \\ h_{0i} &\to h_{0i} + \partial_i \xi_0, \\ h_{ii} &\to h_{ii}, \\ A_0 &\to A_0 + (\partial_0 \xi_0) A_0 + \xi_0 \partial_0 A_0, \\ A_i &\to A_i + (\partial_i \xi_0) A_0 + \xi_0 \partial_0 A_i. \end{aligned} \tag{35}$$

Performing these transformations and promoting $\xi_0 \to \pi$:

$$\mathcal{L}_{\text{int}} = \alpha \left((h_{0i} + \partial_i \pi) A_i - \frac{1}{2} h_{ii} A_0 + A_0 + (\partial_0 \pi) A_0 + \pi \partial_0 A_0 \right) + \delta (h_{00} + 2\partial_0 \pi) A_0. \tag{36}$$

In the limit $M_{pl} \to \infty$, we find (renaming $2\delta \to -\alpha'$)

$$\mathcal{L}_{\text{int}} = \alpha \left(A_i \partial_i \pi + A_0 \right) - \alpha' A_0 \partial_0 \pi. \tag{37}$$

So we can have couplings between $\partial_i \pi$ and the spacial components of a vector A_i as long as we also have a term in the Lagrangian proportional to the time component of the vector A_0. If we go to canonical normalization for the π field, then

$$\mathcal{L}_{\text{int}} = \frac{1}{F^{s-2}} \left(A_i \partial_i \pi + M^2 A_0 \right) - \frac{1}{F'^{s-2}} \left(A_0 \partial_0 \pi \right), \tag{38}$$

where F and F' are two possibly different mass scales, s is the mass dimension of A_μ, and M is the mass from equation (23). Note that the residual space diffeomorphisms forced the first two terms to have the same overall coupling constant.

In the canonical ghost language, we can understand the relationship between these coupling constants by looking at the interactions of the following form, expanding $\phi = M^2 t - \pi$:

$$\begin{aligned} A_\mu \partial^\mu \phi \left(\frac{\partial_\nu \pi \partial^\nu \pi}{M^2} \right)^n &\rightarrow (A_0 M^2 - A_0 \partial_0 \pi + A_i \partial_i \pi) \left(1 - \frac{2n \partial_0 \pi}{M^2} + \cdots \right) \\ &= (A_0 M^2 + A_i \partial_i \pi) - (2n+1) A_0 \partial_0 \pi. \end{aligned} \tag{39}$$

So we see that indeed, the coupling constant of the $A_0 \partial_0 \pi$ term is independent of the other two terms.

6 Coupling to Fermions Currents

Now we would like to couple π to some fermion currents in order to look for Lorentz violating effect in the Standard Model. A natural choice is

$$A^\mu = \sum_\psi c_\psi \bar{\psi} \bar{\sigma}^\mu \psi, \tag{40}$$

where c_ϕ are arbitrary coefficients. If we make the simplifying assumption that $F = F'$, then equation (38) leads to the interaction from [1]. Here, we will start with the assumption $F' = 0$, leading to the interaction

$$\mathcal{L}_{\text{int}} = \sum_\psi \frac{c_\psi}{F} (M^2 \bar{\psi} \bar{\sigma}^0 \psi - \bar{\psi} \bar{\sigma}^i \psi \partial_i \pi). \tag{41}$$

Note however, that we can remove some of these couplings by the field redefinition

$$\psi \rightarrow e^{i c_\psi (M^2 t - \pi)/F} \psi. \tag{42}$$

In that case, the kinetic terms for the ψs become

$$\sum_\psi i \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi \rightarrow \sum_\psi i \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi - \sum_\psi \frac{c_\psi}{F} (M^2 \bar{\psi} \bar{\sigma}^0 \psi - \bar{\psi} \bar{\sigma}^i \psi \partial_i \pi - \bar{\psi} \bar{\sigma}^0 \psi \partial_0 \pi), \tag{43}$$

effectively trading an F term for a F' term. Because the interesting Lorentz violating effects come from the F term, we want to create a Lagrangian that does not have all of the rephasing symmetry.

It will turn out that in order for ghost condensation to make sense as an infrared modification of gravity, the scale M has to be much smaller than the electroweak scale. At this scale, we do have terms in the Lagrangian that break the arbitrary rephasing symmetry, namely Dirac mass terms like $m_D \psi \psi^c$. We can use the remaining diagonal rephasing symmetry to remove the vector coupling, but we will still be left with an axial coupling. In Dirac notation:

$$\mathcal{L}_{\text{int}} = \sum_\Psi \frac{1}{F} (M^2 \bar{\Psi} \gamma^0 \gamma^5 \Psi + \bar{\Psi} \vec{\gamma} \gamma^5 \Psi \cdot \nabla \pi), \tag{44}$$

where we have absorbed the factor of c_Ψ into the definition of F. The first term modifies the quadratic piece of the Lagrangian. The equation of motion for Ψ (*i.e.* the modified Dirac equation) is

$$\left(\gamma^\mu p_\mu - \mu \gamma^0 \gamma^5 - m_D \right) \Psi(p) = 0, \tag{45}$$

where $\mu = M^2/F$. We can calculate the eigenvalues of this equation to find the dispersion relation:

$$\omega = \sqrt{(|k| \pm \mu)^2 + m_D^2}. \tag{46}$$

If we go to the limit $m_D \to 0$, the plus sign corresponds to left-handed particles and anti-particles and the minus sign corresponds to right-handed particles and anti-particles.

Performing a Lorentz boost on the first term in equation (44), we can generate an interaction

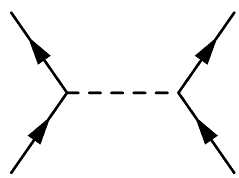
$$\mathcal{L}_{\rm int} = \mu \bar{\Psi} \vec{\gamma} \gamma^5 \Psi \cdot \vec{v}, \tag{47}$$

where $\vec{v}$ is the velocity of our terrestrial experiments with respect to the preferred rest frame of the universe. In the non-relativistic limit, this coupling is equivalent to a spin interaction Hamiltonian

$$H_{\rm int} = \mu \vec{S} \cdot \vec{v}. \tag{48}$$

By performing precision atomic experiments, one can place limits on this coupling by looking for any signals that vary as the earth moves around the sun. (I will discuss more about the limits on Lorentz violating interactions in a follow-up paper.)

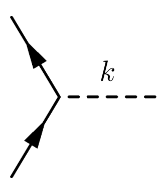
The most surprising interaction comes from the second term in equation (44). It enables us to draw a Feynman diagram involving π exchange.



(49)

If we calculate the amplitude for this process in the non-relativistic limit, then through the Born approximation, the Fourier transform of the amplitude will be proportional to the classical potential. The Feynman rules for π in the non-relativistic limit ($\omega \to 0$) are

$$= \frac{i}{w^2 - \frac{\tilde{M}^2}{M^4} k^4 + \frac{\tilde{M}_3^3}{M^4} w k^2} \underset{\omega \to 0}{\longrightarrow} \frac{-iM^4}{\tilde{M}^2} \frac{1}{k^4}, \tag{50}$$



$$= \frac{1}{F} \vec{\gamma} \gamma^5 \cdot \vec{k}. \tag{51}$$

The amplitude is therefore proportional to

$$\mathcal{M} \sim \frac{M^4}{\tilde{M}^2 F^2} \frac{(S_1 \cdot \vec{k})(S_2 \cdot \vec{k})}{k^4}. \tag{52}$$

We can do a Fourier transform of the amplitude to find the classical potential:

$$V(r) \sim \frac{M^4}{\tilde{M}^2 F^2} \frac{\vec{S}_1 \cdot \vec{S}_2 - (\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})}{r}. \tag{53}$$

This looks a lot like magnetic dipole-dipole interactions in electromagnetism (modulo a factor of 3), expect that the force decays as $1/r$ instead of $1/r^3$. This comes entirely from the fact that the dispersion relationship for our π field goes as $w \sim k^2$ instead of the usual $w \sim k$.

In fact, this dispersion relationship means that we need to be a bit careful by what we mean by the non-relativistic limit. If we set up two large sources of spin a distance r apart, then it will take a time

$$\tau \sim \frac{1}{\omega} \sim \frac{M^2}{\tilde{M}k^2} \sim \frac{M^2}{\tilde{M}} r^2 \tag{54}$$

for the π fields to mediate information between source 1 and source 2. For example, if $r = 1$ m, and $M = \tilde{M} = 1$ MeV, then $\tau \sim 5$ hr. This is how long we would have to wait until we would really be able to measure this long-range spin-dependent force. Because M controls both the strength of the interaction and the time-scale over which an experiment would have to take place, it would be very difficult to see this novel interaction if M were either too big or too small.

7 Prospects

We have seen that any theory that gives rise to spontaneous time diffeomorphism breaking has low energy interactions governed by the π field. Like massive gravity, ghost condensation presents us with an interesting infrared modification of gravity, but unlike Fierz-Pauli massive gravity, we end up with a theory that no longer has Lorentz symmetry. We saw that if the ghost coupled directly to the Standard Model, then there was necessarily a relationship between an interaction that modified the dispersion relationship for Dirac fermions, and an interaction that gave rise to a long-range spin-dependent potential.

Our next goal is to understand the experimental bounds on the various parameters of our theory. Physicists have been searching for Lorentz violating effects for a very long time, and the bounds are very stringent. There is a universal language developed by V. A. Kostelecký and friends [5] that parametrizes the space of Lorentz violations. By comparing this model with known bounds, we will be able to test the experimental viability of all theories of spontaneous time diffeomorphism breaking, including ghost condensation.

References

[1] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty, and S. Mukohyama. *Ghost Condensation and a Consistent Infrared Modification of Gravity*. `hep-th/0312099`.

[2] N. Arkani-Hamed, P. Creminelli, S. Mukohyama, and M. Zaldarriaga. *Ghost Inflation*. `hep-th/0312100`.

These are the original ghost papers. [1] starts with the idea of condensing a scalar ϕ with a wrong sign kinetic term à la the Higgs mechanism. [2] uses the field ϕ as a candidate for the inflaton and shows that the experimental signatures are much different than for slow-roll inflation. Both are must reads for anyone interested in spontaneous time diffeomorphism breaking.

[3] N. Arkani-Hamed, H. Georgi, and M. D. Schwartz. *Effective Field Theory for Massive Gravitons and Gravity in Theory Space*. `hep-th/0213184`.

[4] J. Thaler. *Gravity and Unitarity*. `http://www.jthaler.net/physics/notes/GravityUnitarity.pdf`.

These papers deal with aspects of massless and massive gravity. In [3], the authors build the Fierz-Pauli theory of massive gravity by creating an auxilliary diffeomorphism symmetry. [4] is a far more pedestrian overview of what makes gravity a unitary theory at low energies.

[5] D. Colladay and V. A. Kostelecký. *Lorentz-Violating Extension of the Standard Model*. `hep-ph/9809521`.

One of a number of papers exploring how to parametrize Lorentz violations in the Standard Model. More articles in this vein will be cited in an upcoming talk on ghost condensation and Lorentz violation.