The Degenerate MSSM

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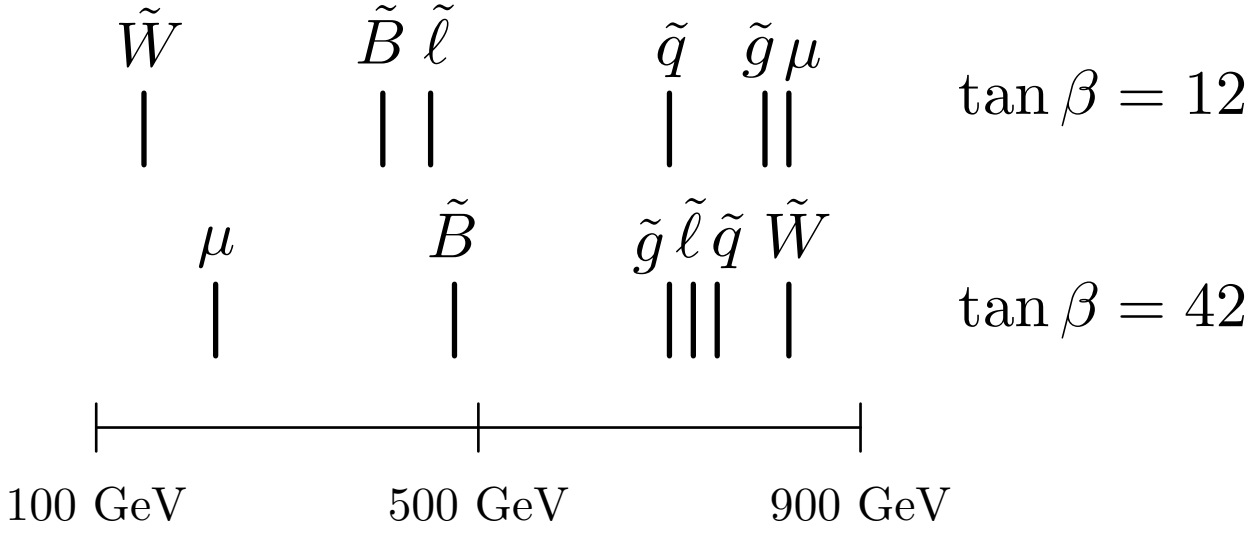
One Year at the LHC

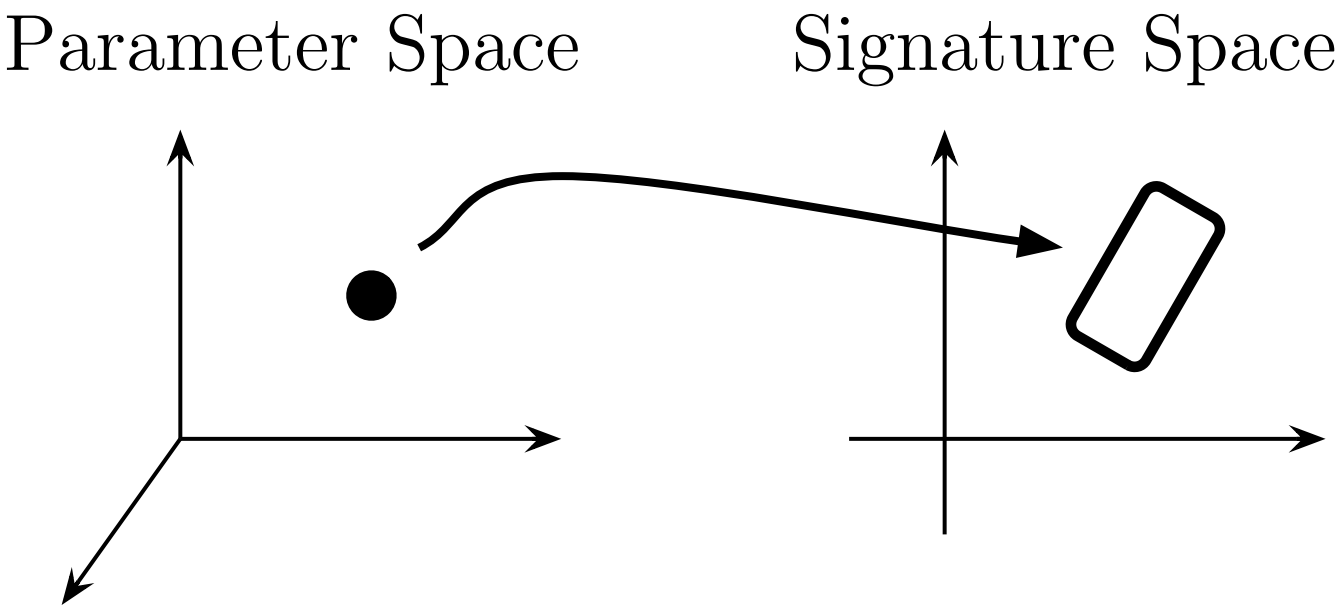
$10\ \text{fb}^{-1}$

Experimental Data $\longrightarrow$ Theoretical Models?

If it's the MSSM...



Standard Method

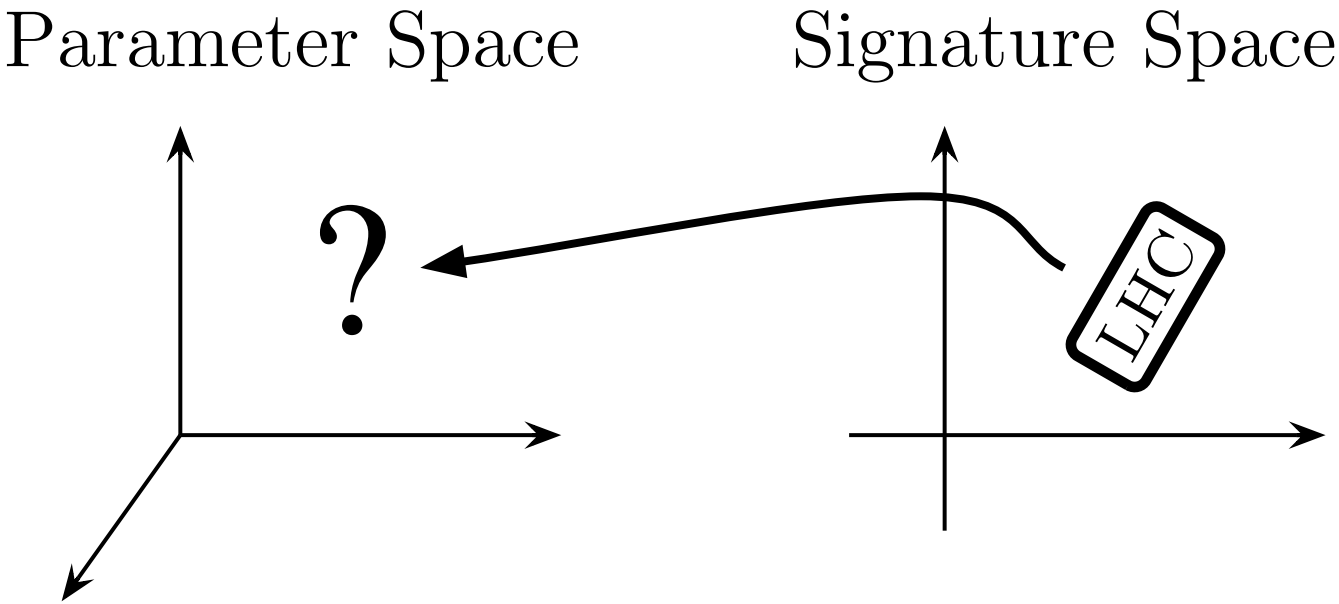


TDR, Benchmark studies, LEP/Tevatron bounds, etc.

Reduced parameter set (mSUGRA, AMSB, GMSB):

Experimental Data $\longrightarrow$ Precision Measurements

The Inverse Problem

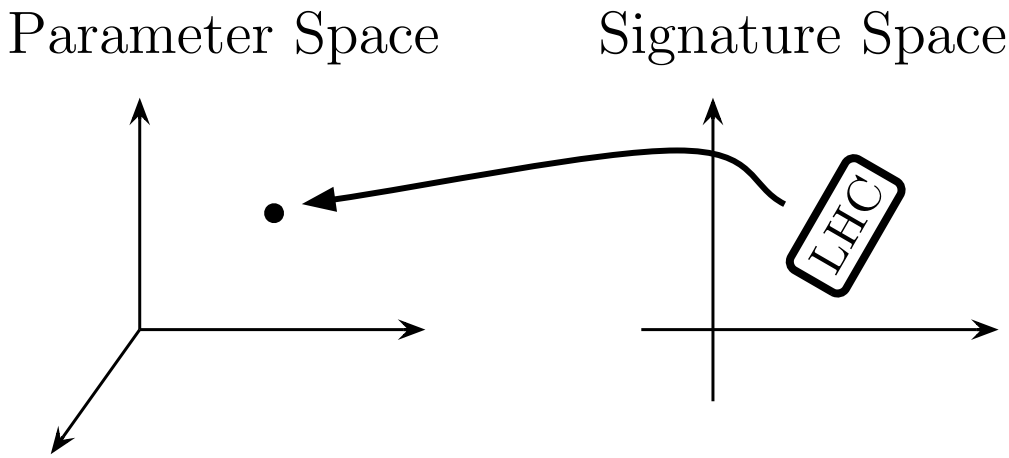


Much more interesting!

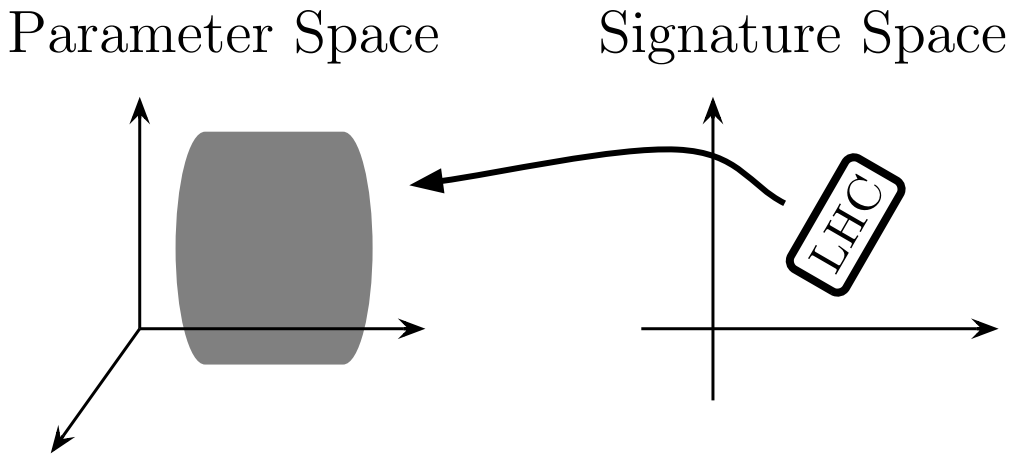
$$\text{Data} \xrightarrow{?} \text{gaugino unification, dark matter, ...}$$

Much more important! (500 GeV ILC in 10 years?)

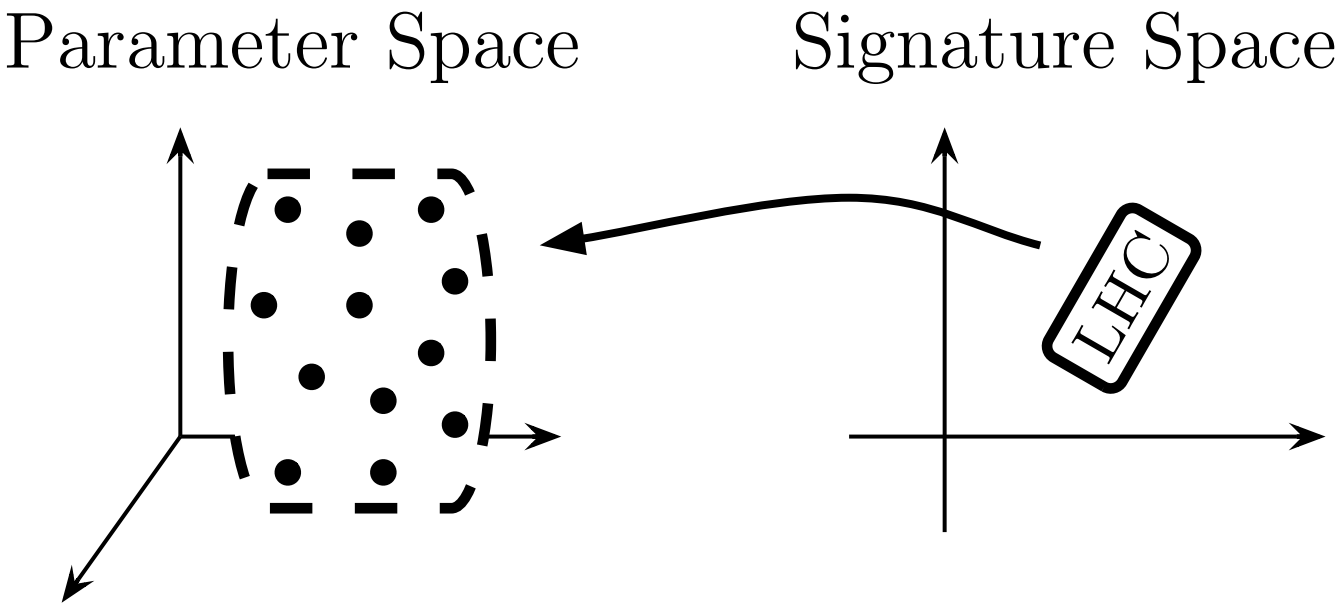
Best of all Possible Worlds



Worst of all Possible Worlds



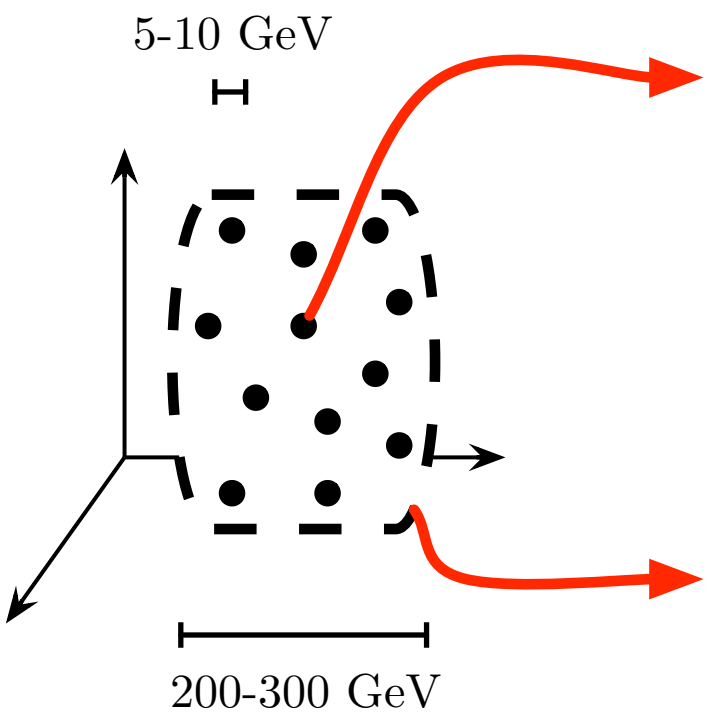
The Real MSSM



Degeneracies!

Many small footprints in a large overall region.

The Degenerate MSSM

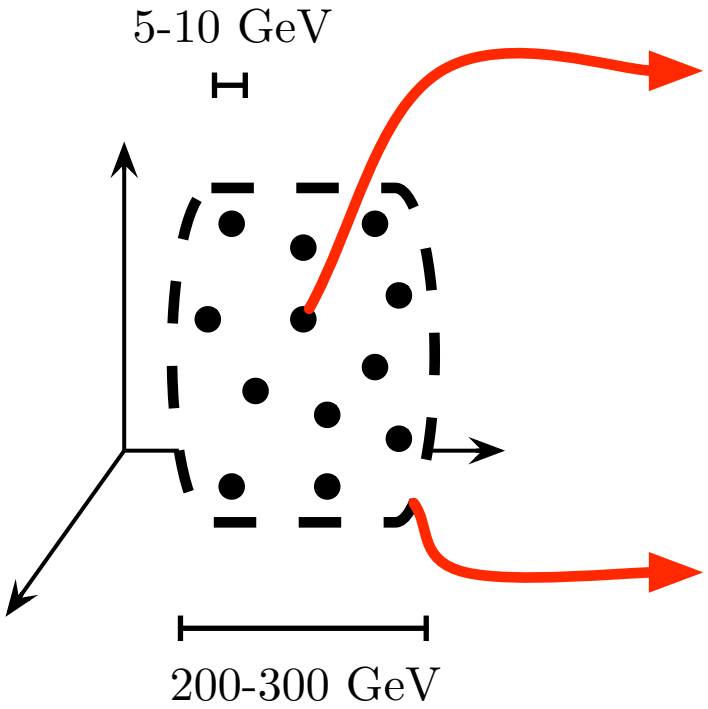


If LHC data is consistent with mSUGRA, you will measure mSUGRA parameters very accurately...

...but there at least $\mathcal{O}(100)$ other (well-motivated?) MSSMs consistent with the same experimental data.

200 GeV can change gaugino unification, LSP identity, etc.

On The Snowmass Slopes

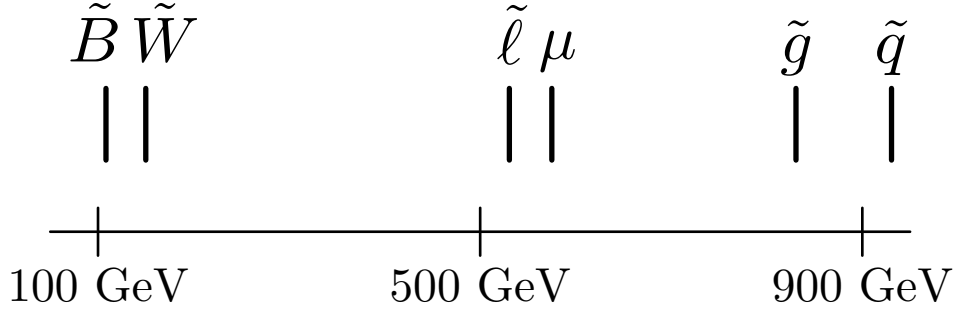


Slope 1a: (Allanach, et al, 2002.)

$m_{1/2} = 425$ GeV

$m_0 = -A_0 = 170$ GeV

$\mu > 0, \quad \tan\beta = 10$



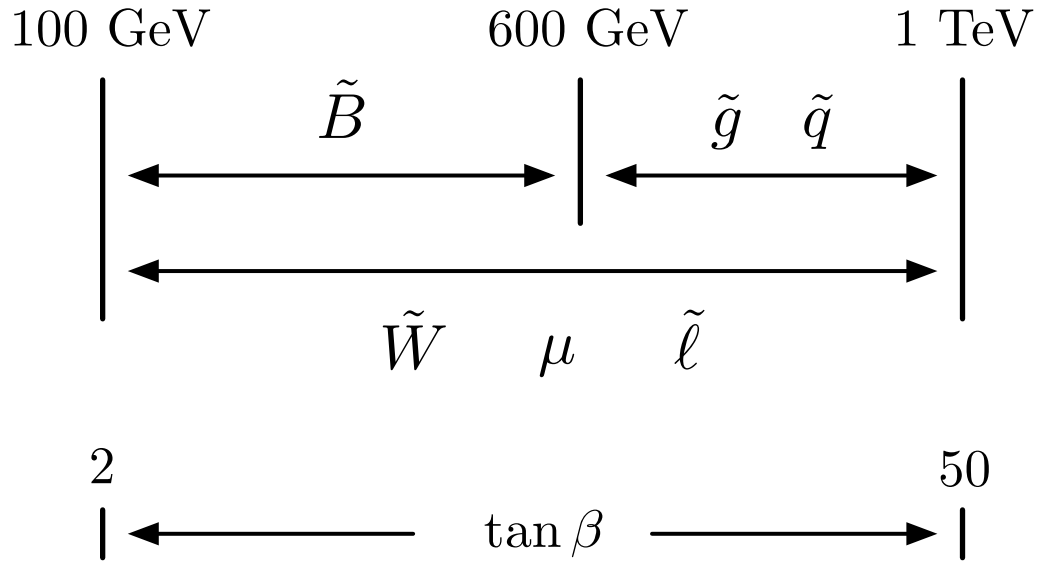
$\tan\beta = 46$

$\chi^2_{\rm sig} = 5$

Strategy

50,000 MSSMs

Nowhere near full coverage, no flavor dependence.



No slepton LSP. Everything else at PYTHIA default values.

Inclusive Signatures

$$2 \text{ jets} + \not{E}_T + \begin{cases} 0\ell \\ 1\ell \\ \text{OS} \\ \text{SS} \\ 3\ell \\ 4^+\ell \end{cases} \qquad \begin{array}{ll} & \langle \not{E}_T \rangle \\ \text{jet multiplicity} & \langle E_T^{\text{lep}} \rangle \\ b\text{-jet multiplicity} & \langle E_T^{\text{jet}} \rangle \end{array}$$

(Baer, Chen, Paige, Tata, 1995.)

Total of 22 (very broadbrush) signatures.
Real LHC observables.

Standard TDR-like cuts. No SM background.
Assume 10% error.

What can you do with 50,000 MSSMs?

$$(50000)^2 \text{ Pairs!}$$

$$\downarrow$$

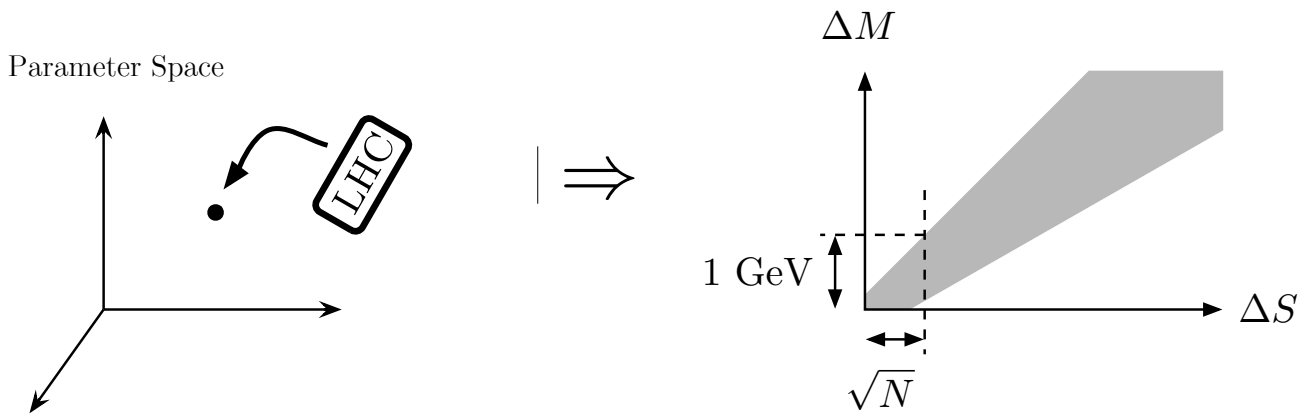
Statistics!

For each pair, signature vs. parameter distance.

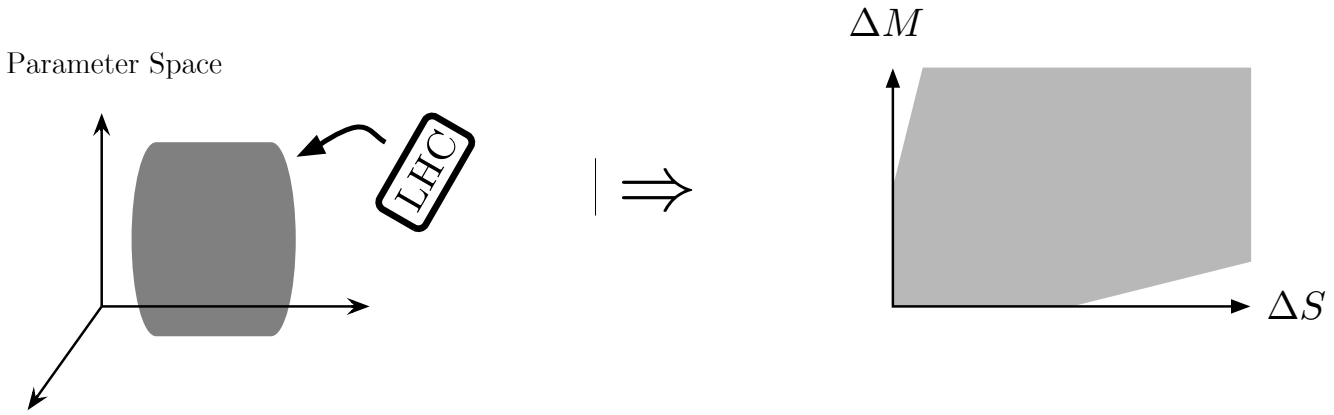
$$\Delta S = \chi^2_{\text{sig}} = \sum_i \left(\frac{\Delta s_i}{\sigma_i} \right)^2$$

$$\Delta M = \text{“}\chi^2_{\text{para}}\text{”} = \sum_j \left(\frac{\Delta m_j}{\text{“}\sigma_j\text{”}} \right)^2$$

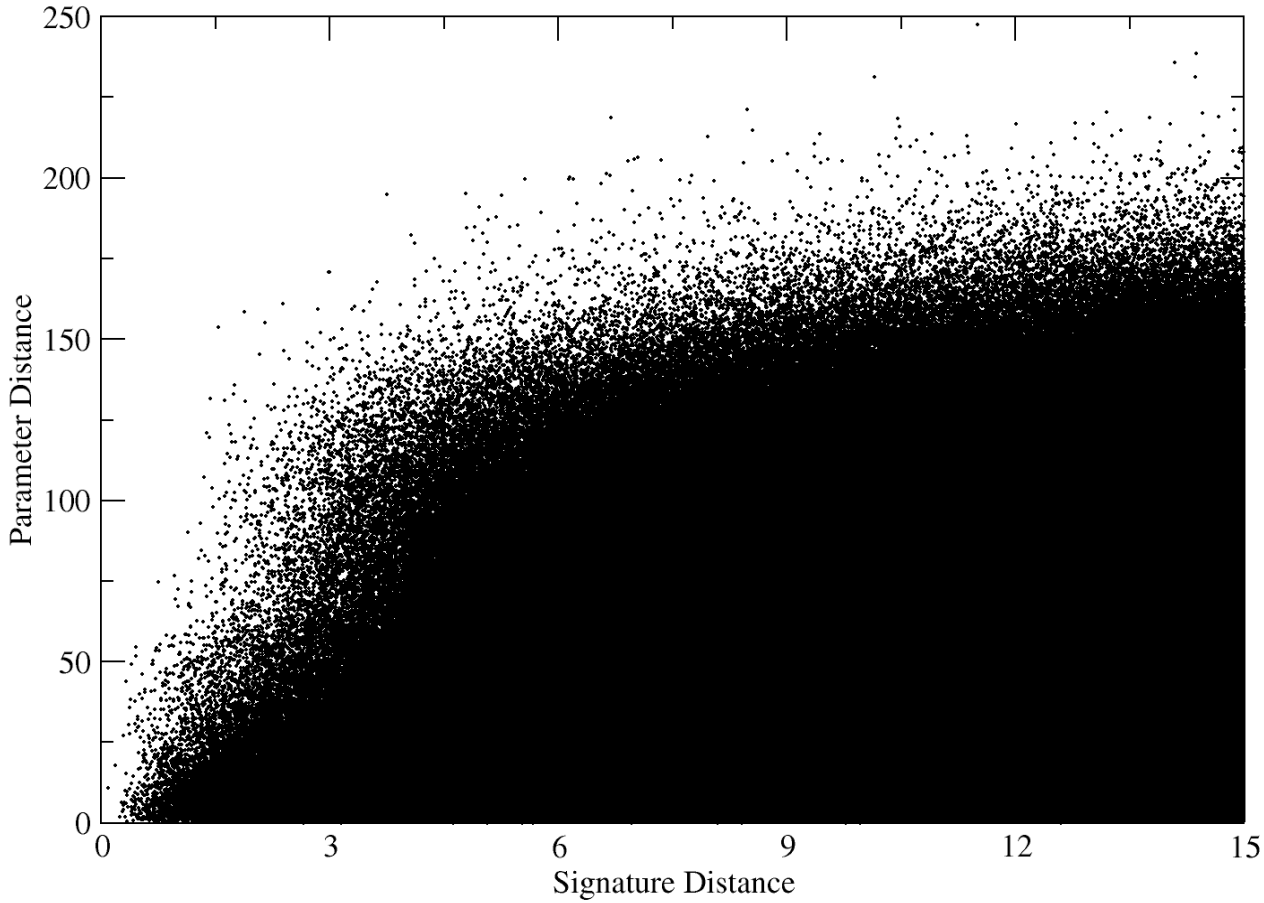
Best of all Possible Worlds



Worst of all Possible Worlds

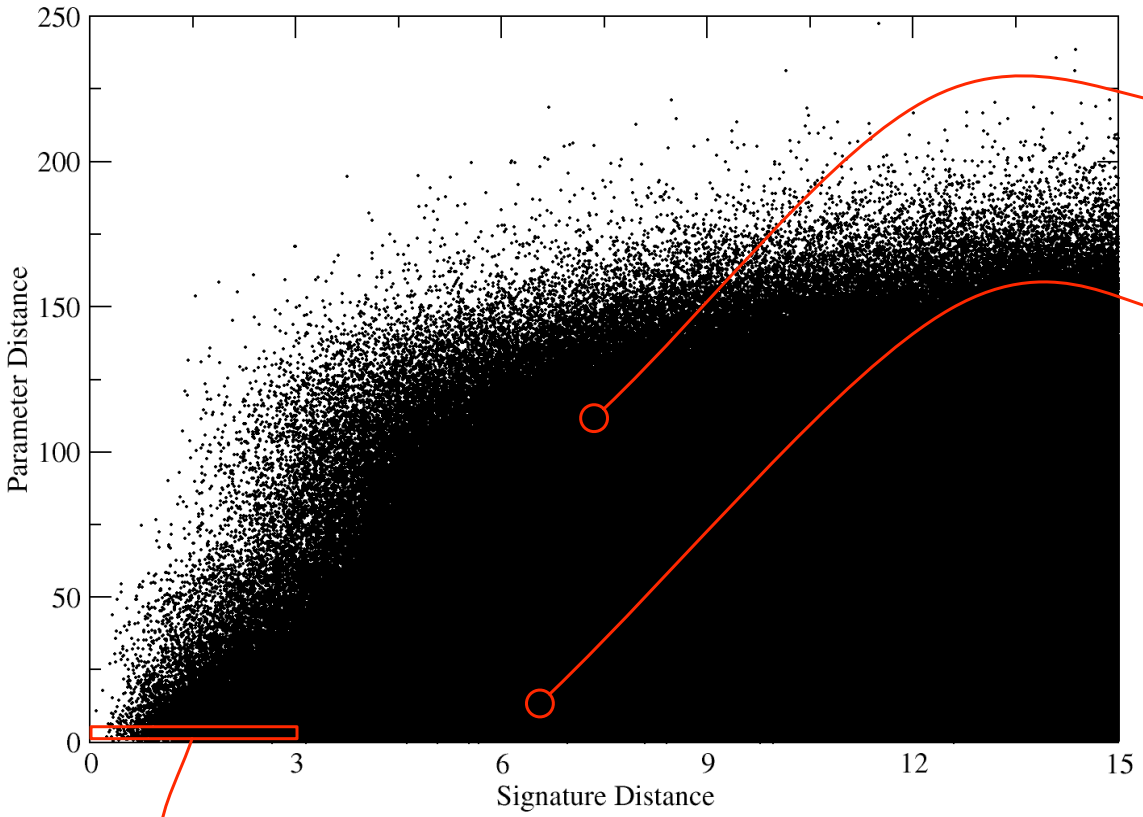


The Real MSSM



Shown: 4.9×10^6 of 2.6×10^9 pairs.

The Real MSSM



MSSM 2005 vs.
MSSM 22725

MSSM 2005 vs.
MSSM 47553

LHC Success Region

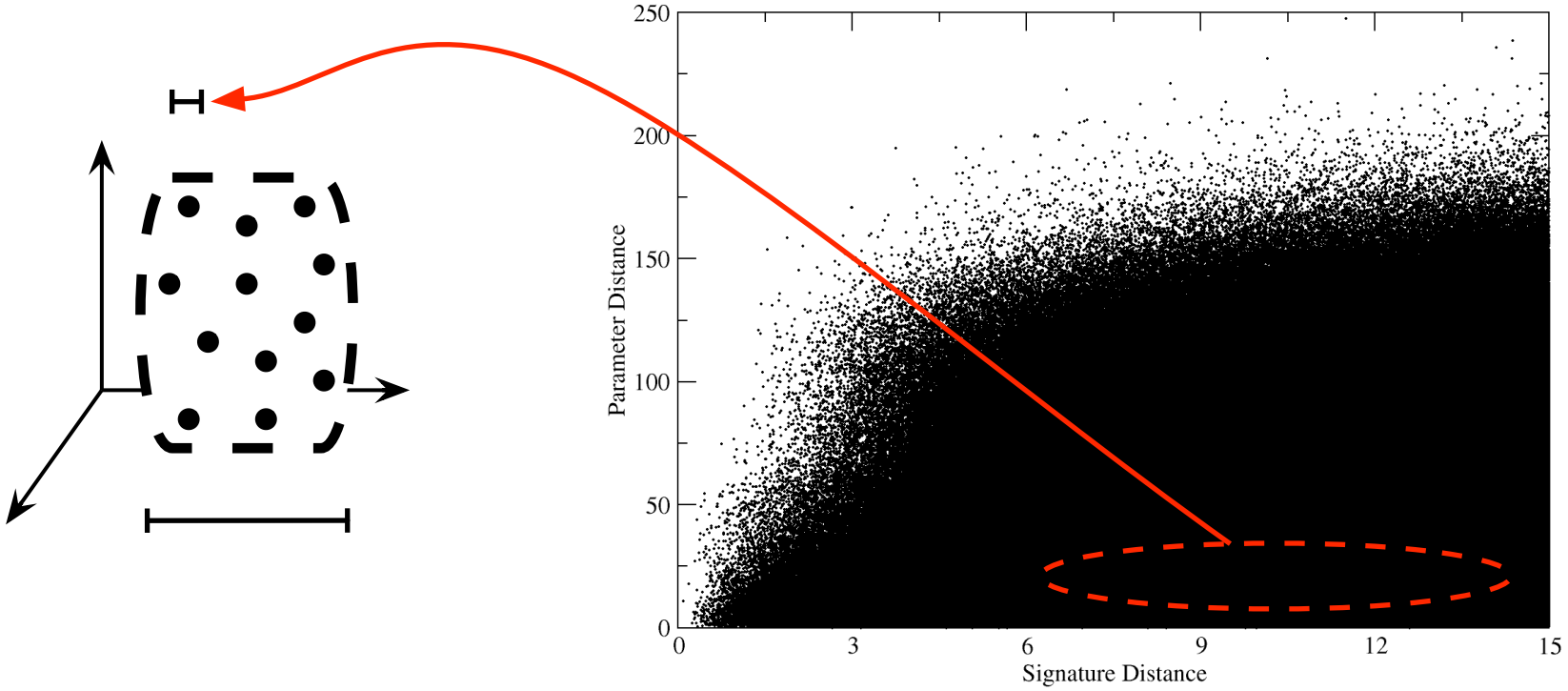
$$\begin{cases} \chi^2_{\text{para}} < .75 \leftrightarrow 5\% \text{ error} \\ \chi^2_{\text{sig}} < 3.0 \leftrightarrow 1.5 \times \sqrt{N} \text{ error} \end{cases}$$

MSSM 2005

```
mephisto:~/Pythia jthaler$ ./easyinfomod 2005

*******

  MSSM  ERRMASS     bino     wino   gluino       mu   squark  slepton tan beta
  2005     0.00     204.     909.     778.     588.     661.     662.      47.
 47553    18.94     149.     771.     621.     460.     733.     979.      45.
 22725   108.88     562.     226.     613.     994.     782.     841.      41.

*******

  MSSM  ERRSIGS       0l    xj    bj       1l    xj    bj       OS    xj    bj
  2005     0.00    31202  2.21  0.17      506  2.14  0.35       25  2.00  0.65
 47553     6.43    30755  2.56  0.19      587  2.32  0.28       22  2.17  0.78
 22725     7.24    26106  2.53  0.19      397  2.27  0.37       13  1.93  0.57

  MSSM  ERRSIGS     lepETmed   misETmed   jetETmed
  2005     0.00           84.       286.       305.
 47553     6.43           93.       264.       270.
 22725     7.24           86.       259.       259.
```

Two Surprises: Cliffs

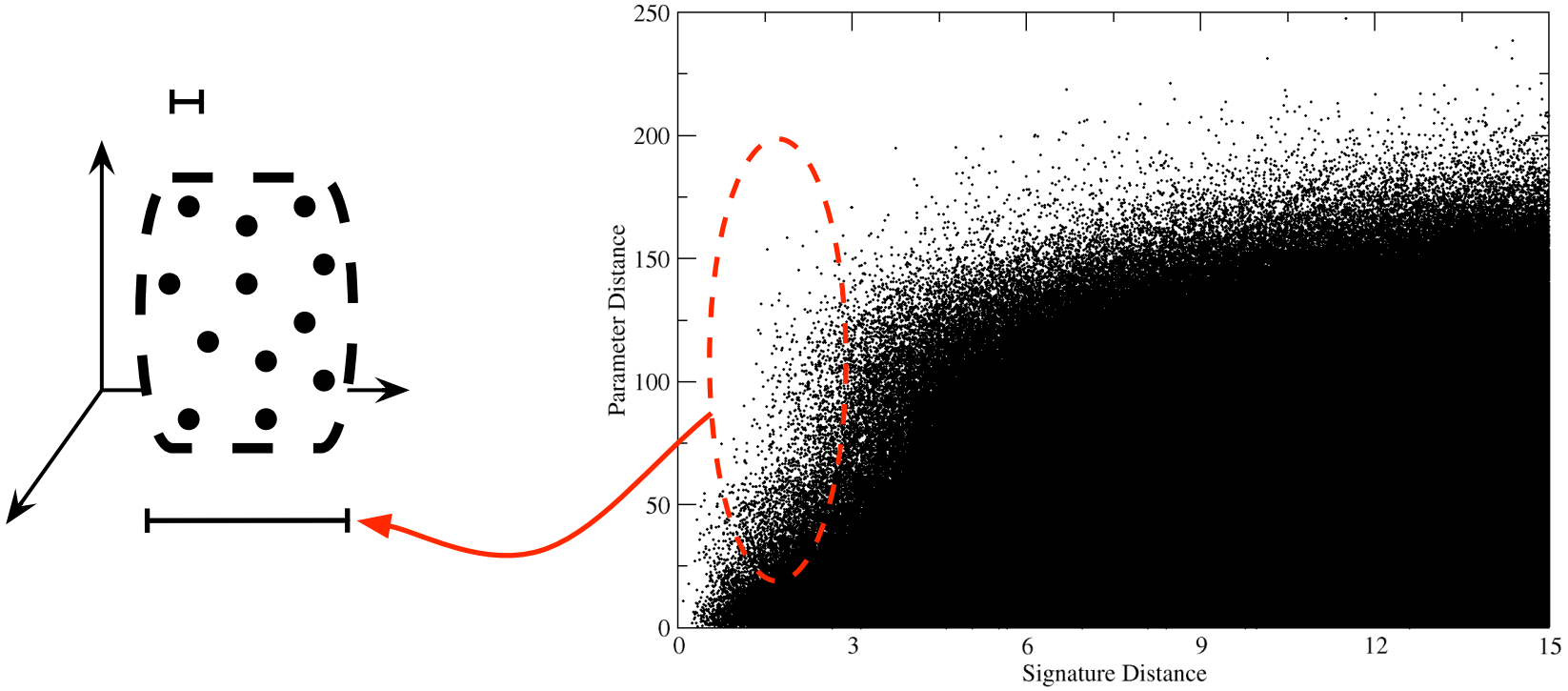
Close in parameter space, far away in signature space.



Evidence for small individual footprint size.

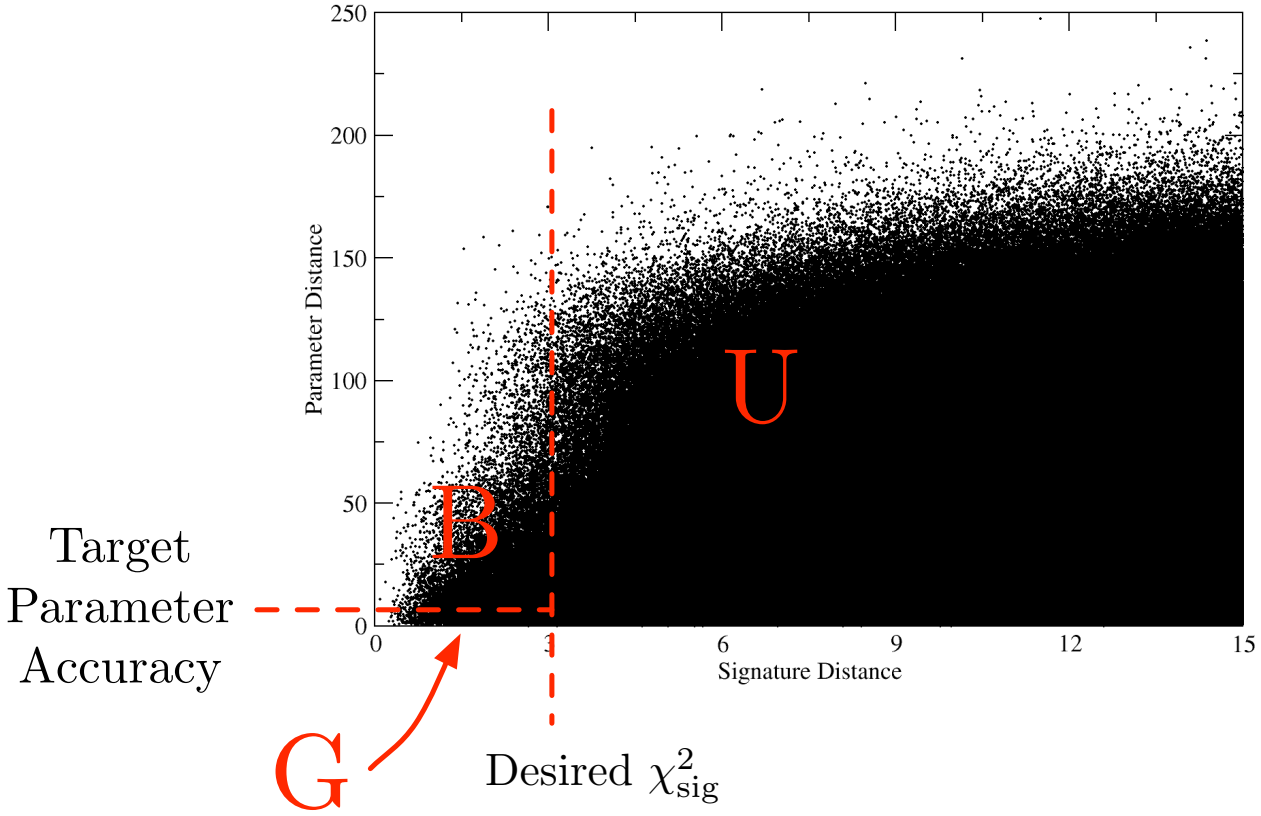
Two Surprises: Degeneracies

Close in signature space, far away in parameter space.



Evidence for large number of footprints.

Counting Degeneracies



$$\langle d \rangle = \frac{G+B}{G} \sim 100\ (!) \qquad \chi^2_{\rm sig} < 3 \qquad \Delta M \lesssim 5\%$$

Why So Many Degeneracies?

$$N_{\text{MSSMs}} \;\overset{?}{\gtrless}\; N_{\text{sig bins}}$$

N_{MSSMs} $\uparrow$ different MSSMs measured in desired parameter accuracy

$N_{\text{sig bins}}$ $\uparrow$ distinguishable experimental outcomes at the LHC

Pigeon hole principle!

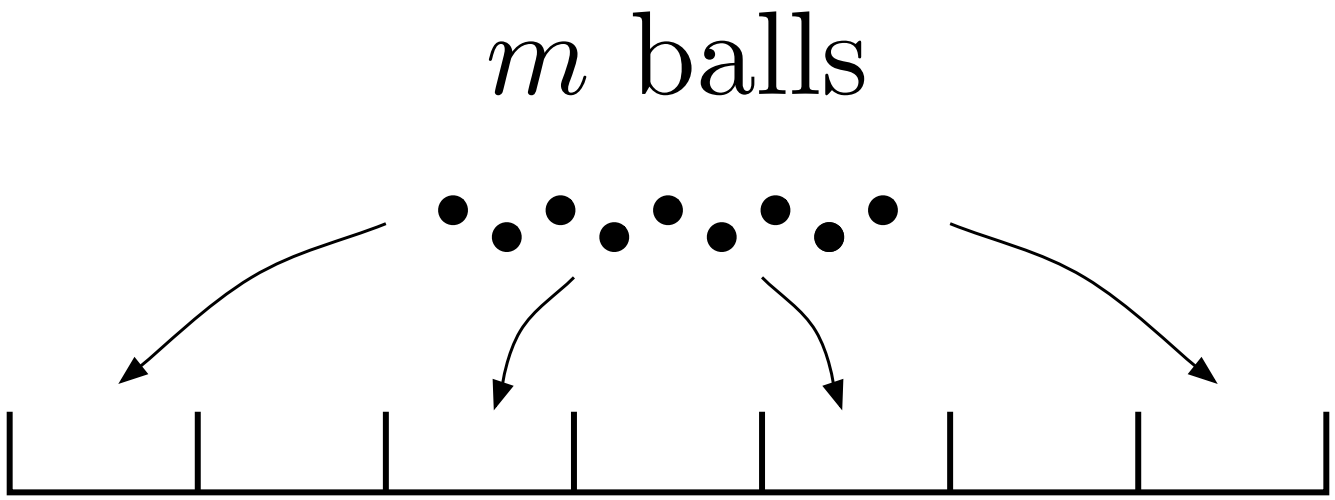
5% bins in parameter space (modulo decoupling):

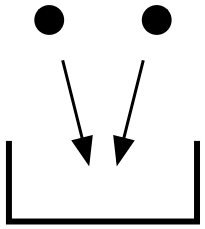
$$N_{\text{MSSMs}} \sim 10^7$$

Simple way to count signature bins for given χ^2_{sig}:

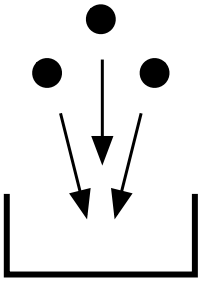
$$N_{\text{sig bins}} = \begin{cases} 310{,}000 & \chi^2_{\text{sig}} < 2 \\ 83{,}000 & \chi^2_{\text{sig}} < 3 \\ 5{,}000 & \chi^2_{\text{sig}} < 6 \\ 1{,}600 & \chi^2_{\text{sig}} < 9 \\ 800 & \chi^2_{\text{sig}} < 12 \\ 400 & \chi^2_{\text{sig}} < 15 \end{cases}$$

Statistics



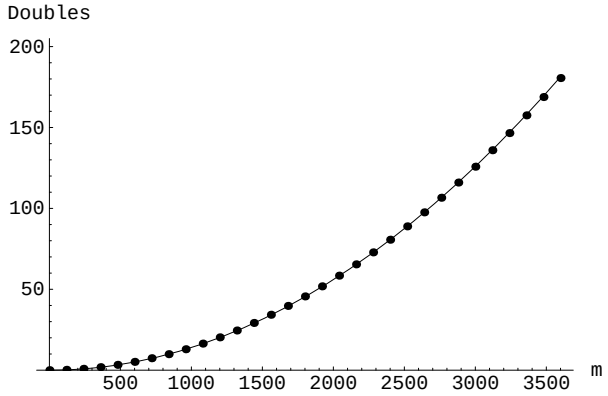


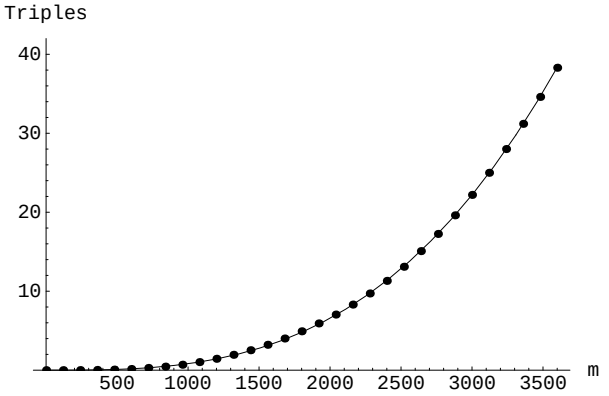
$$= \frac{1}{2!}\frac{m^2}{N}$$

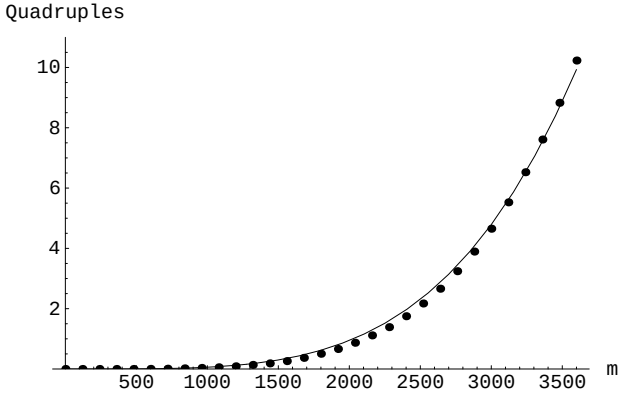


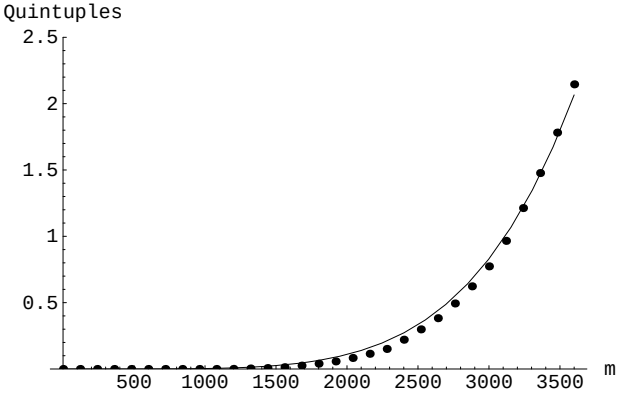
$$= \frac{1}{3!}\frac{m^3}{N^2}$$

Statistics of the MSSM









$$N_{\text{sig bins}} \sim 83{,}000 \qquad \chi^2_{\text{sig}} \lesssim 3$$

Why So Many Degeneracies? (Redux)

$$D_{\text{parameters}} \overset{?}{\geq} D_{\text{signatures}}$$

$D_{\text{parameters}}$: number of independent MSSM parameters

$D_{\text{signatures}}$: number of independent signature directions

$$D_{\text{parameters}} = 7 \qquad D_{\text{signatures}} = 22?$$

Correlation Matrix

Are inclusive signatures independent?

$$C_{ij} = \frac{\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle}{\sigma_i \sigma_j} \qquad \sigma_i = \langle s_i^2 \rangle - \langle s_i \rangle^2$$

Assuming linear relations:

$$\text{Number of } \lambda_C > 1 \simeq D_{\text{signatures}}$$

$$\text{eigenvalues}(C) = \{6.7, 4.5, 2.4, 1.8, 1.3, < 1.0, \ldots\}$$

For $D_{\text{parameters}} = 7$:

$$D_{\text{signatures}} \simeq 5$$

Signature Distance

$$|s| = \sqrt{\chi^2_{\text{sig}}}$$

How does number of signature bins scale with $|s|$?

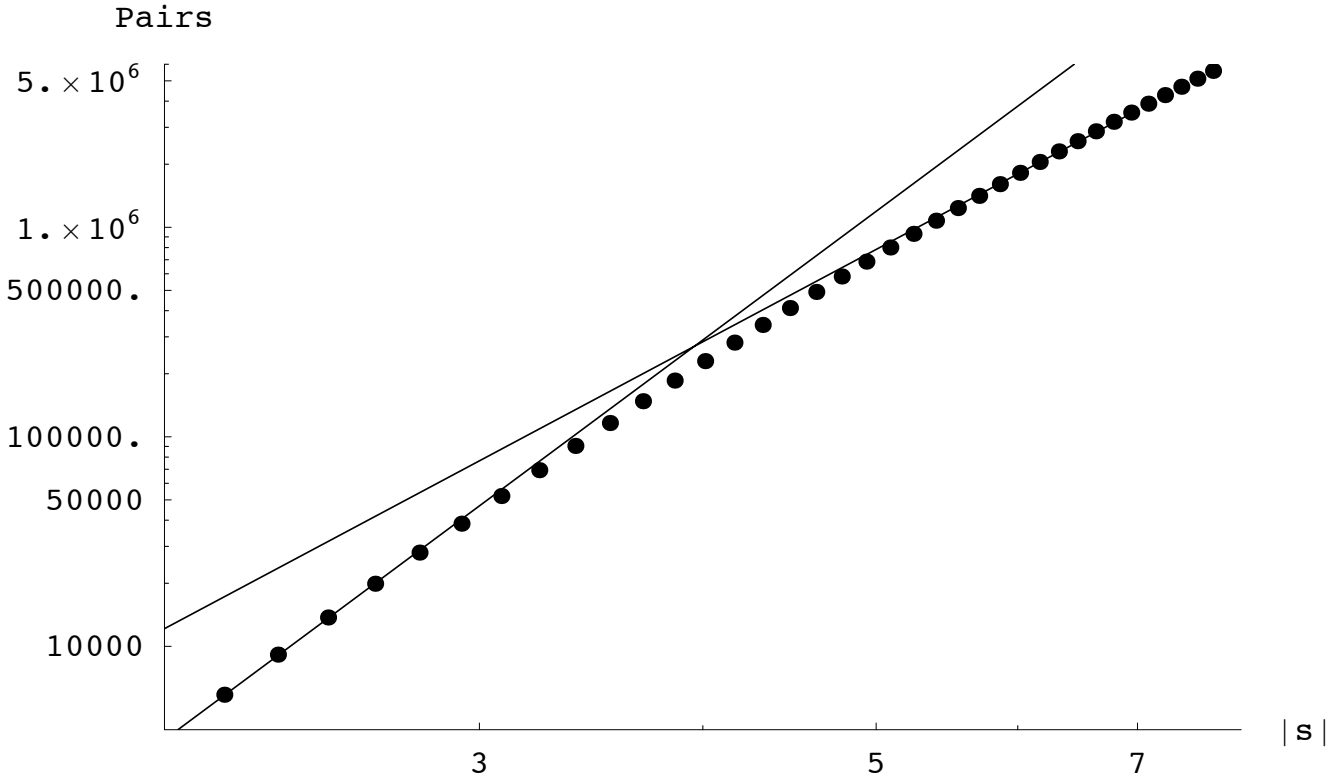
$$N_{\text{sig bins}} \sim |s|^{-D}$$

Or equivalently...

$$\#\text{ pairs closer than } |s| \sim |s|^{D}$$

(A measure of how signature volume scales with radius.)

Scaling of Pairs



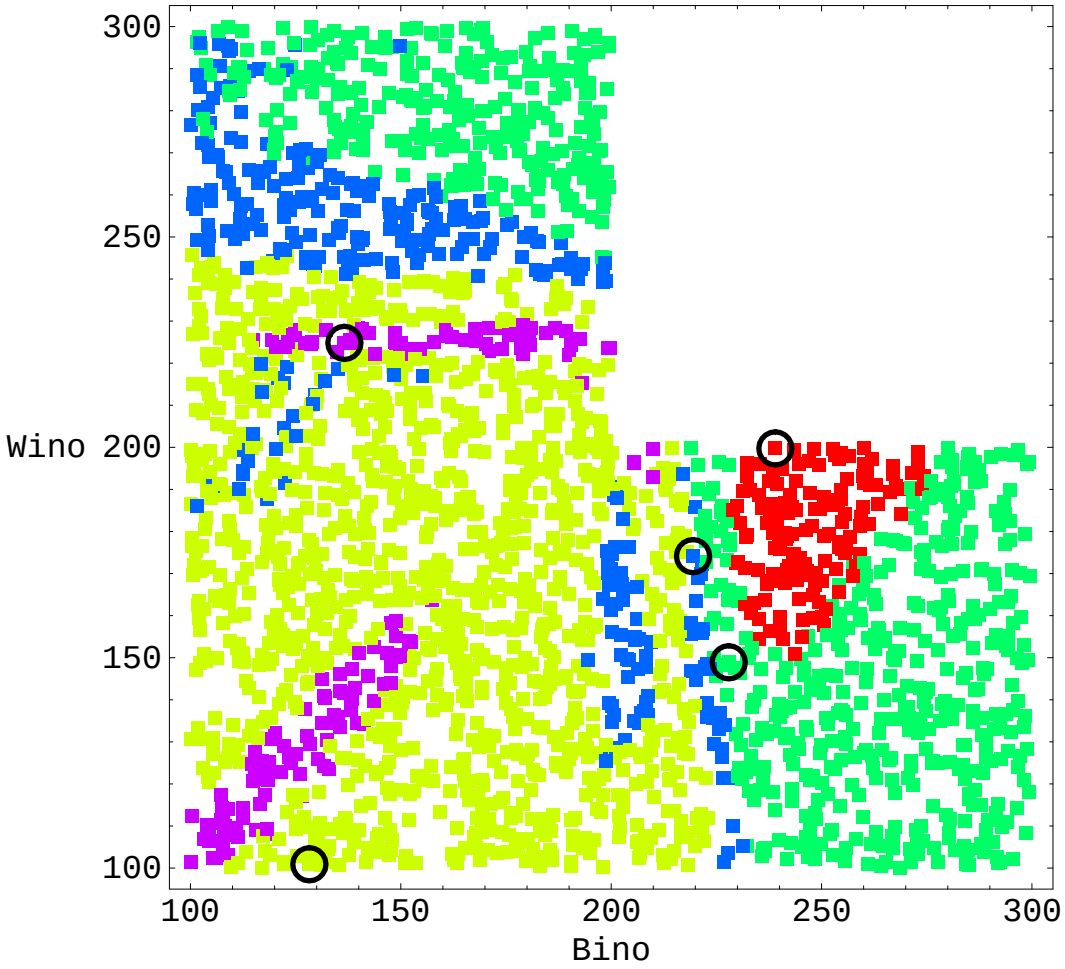
$$6.3 \gtrsim D_{\text{signatures}} \gtrsim 4.5$$

Are There Really Degeneracies?

Proof by example. 2500 MSSMs in the range:

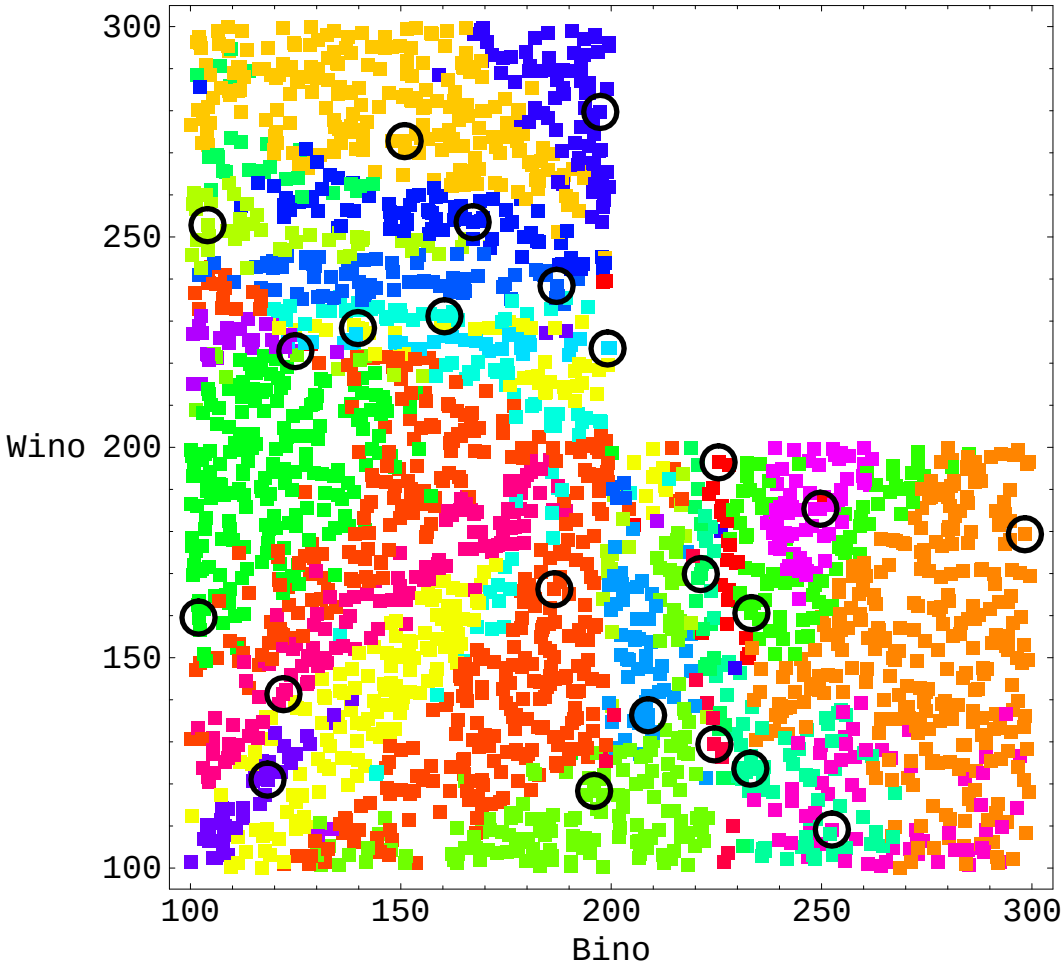
$\tilde{B}$	100 GeV – 300 GeV
$\tilde{W}$	100 GeV – 300 GeV
$\tilde{g}$	793 GeV
μ	294 GeV
$\tilde{q}$	845 GeV
$\tilde{\ell}$	207 GeV
$\tan\beta$	11

All points have 10,000 – 16,000 SUSY events that pass TDR-like cuts. Plenty of leptons.



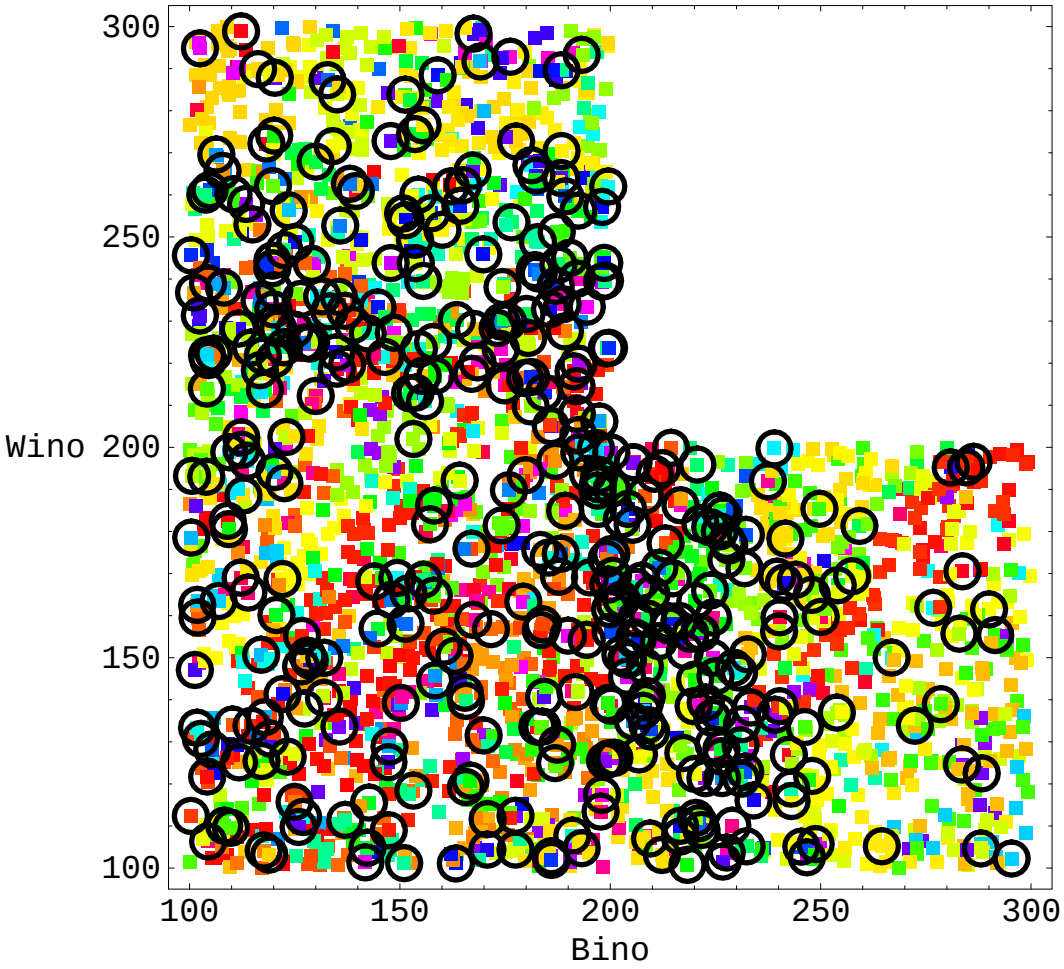
$\chi^2_{\rm sig} < 45$

All yellow points are $\chi^2_{\rm sig} < 45$ of the circled yellow point.



$\chi^2_{\rm sig} < 15$

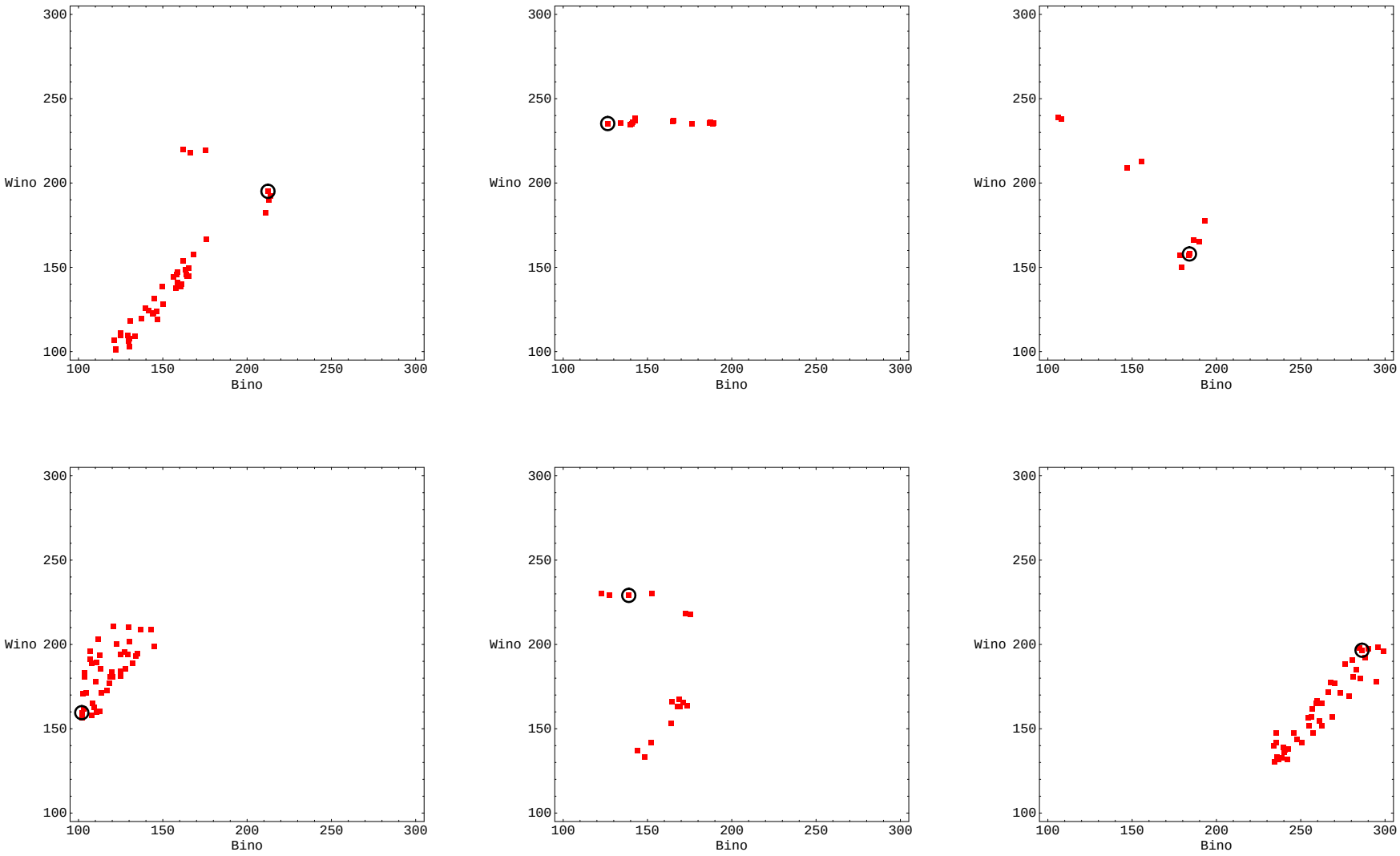
23 regions.



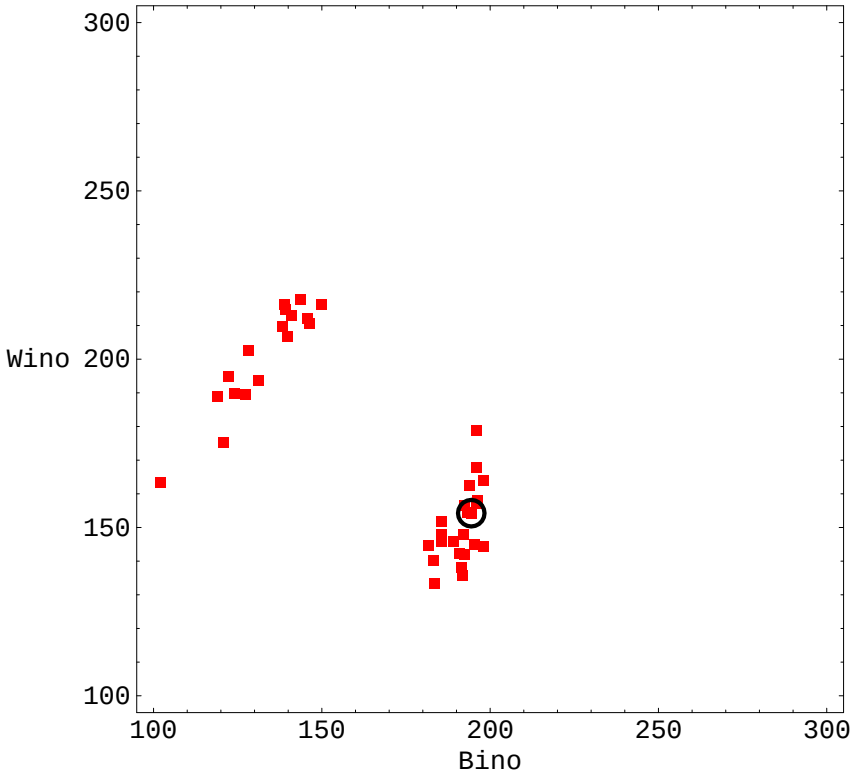
$$\chi^2_{\text{sig}} < 4$$

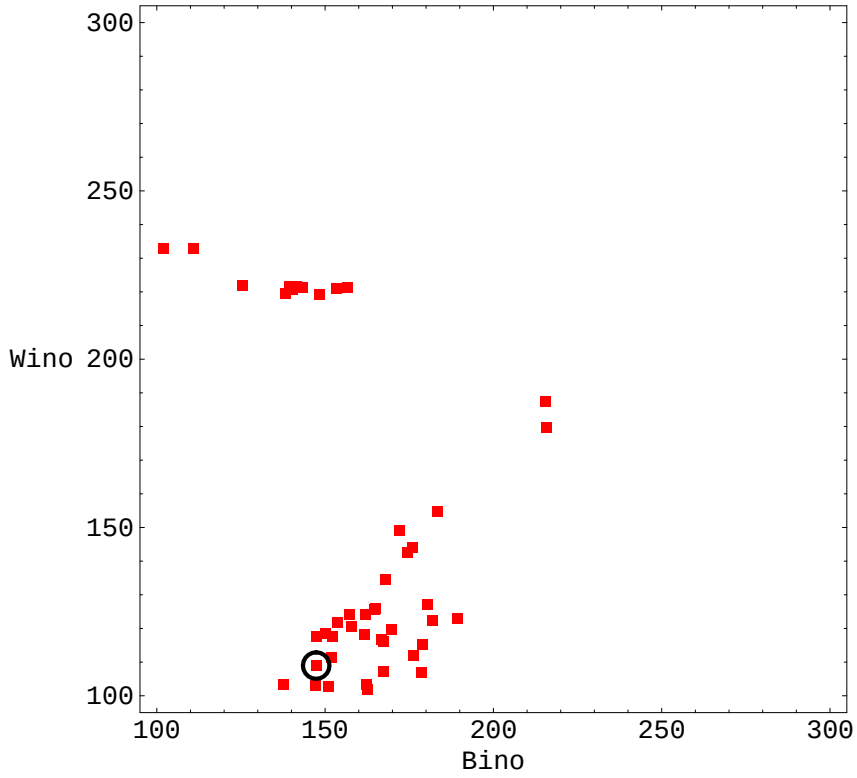
416 regions. Lots of fragmention, but...

Degeneracy City



LSP Identity? Gaugino Unifcation?





What inclusive signatures can break these degeneracies?

Reconciling Two Different Attitudes

SUSY at LHC is easy!

> If a constrained model is consistent with LHC data, then inclusive signatures can yield precision measurements at 10 fb^{-1}.

SUSY at LHC is hard!

> There are $\mathcal{O}(100)$ different models that also match the data, and there is no systematic way to find them. Do they come from nice UV theories?

Reality Checks

Qualitatively the same story if we:

- Use $\sqrt{N}$ error.
- Enforce > 10000 SUSY events.
- Artificially include dilepton edges/endpoints.

Including flavor information:

- 7 Parameters $\rightarrow$ 14 Parameters
- $10^7 \rightarrow 10^{14}$ MSSMs
- $10^5 \rightarrow 10^6$ Signature Bins

Even more degeneracies!

Lessons from the Degenerate MSSM

LHC Signature Space

$\downarrow$

MSSM Parameter Space

To nail the MSSM at the LHC, we must drastically increase the number of independent inclusive signatures.

Easy to check if a new inclusive works: calculate $\langle d \rangle$.

MSSM vs. NMSSM vs. UED vs. T-parity LH ?