Regarding α_2^{PPN}

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1 Should we trust gr-qc/0412086?

Putting aside the question of whether it even makes sense to consider the hydrodynamics of the sun, I am certain that the type of analysis done in gr-qc/0412086 is identical to the PPN formalism. All of the relevant fields are expanded in spherical harmonics, and far away from the source, the leading effects can be captured by $\hat{r} \cdot \hat{v}$ type effects. What makes me a bit suspicious is that the final effect they find is proportional to v whereas the PPN effects are usually proportional to v^2, which either means that they made a mistake, or the effect they are finding requires that they are in a "shockwave" limit where the direction of radiation matters. In any case, we know from direct computation that no effect depends on the *sign* of v for $v \ll v_c$, so I doubt that their finding are relevant for gauged ghost condensation when $v \ll v_c$.

But to give gr-qc/0412086 the benefit of the doubt, I tried bounding α_2^{PPN} by the deviation of Mercury from the solar ecliptic. This gives a very nice naive bound for α_2^{PPN} which is close to the known experimental limit. Interestingly, if I use incorrect reasoning, I am able to get a bound of comparable magnitude to their bound, which may or may not explain why their bound is erroneously strong.

2 Heuristic Bound on α_2^{PPN}

Consider the modified Newtonian potential with α_2^{PPN} turned on:

$$V(r) = -\frac{GM_\odot m}{r}\left(1 + \frac{\alpha_2^{PPN} v^2}{2}\left((1 - (\hat{r}\cdot\hat{v})^2\right)\right), \tag{1}$$

where $v \sim 10^{-3}$. For numerical reasons, I will work with a slightly different potential

$$V(r) = -\frac{GM_\odot m}{r}\left(1 - \frac{\alpha_2^{PPN} v^2}{2}(\hat{r}\cdot\hat{v})^2\right), \tag{2}$$

which differs from the strict α_2^{PPN} modification by a redefinition of G, assuming v is constant.

Using spherical coordinates, let

$$\vec{v} = (v \sin\theta_v, 0, v\cos\theta_v)\,, \qquad \vec{r}(t) = (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta)\,, \tag{3}$$

where r, θ, and ϕ are functions of t. The lagrangian of the system is

$$L = \frac{1}{2}m\dot{\vec{r}}^2 + \frac{GM_\odot m}{r}\left(1 - \frac{\alpha_2^{PPN} v^2}{2}(\hat{r}\cdot\hat{v})^2\right), \tag{4}$$

from which it is trivial (but tedious) to derive the equations of motion. Expanding r, θ, and ϕ in the parameter $c = \alpha_2^{PPN} v^2$, we can easily solve the equations of motion to leading order in c. We will assume a perfectly circular orbit in the $\theta = \pi/2$ plane at $t = 0$. It is convenient to express θ as a function of ϕ:

$$\phi(t) = \frac{\sqrt{GM_\odot}}{r_0^{3/2}}t + \mathcal{O}(c), \qquad r(t) = r_0 + \mathcal{O}(c), \qquad \theta(\phi) = \frac{\pi}{2} + \frac{c\sin 2\theta_v}{4}\phi\sin\phi + \mathcal{O}(c^2). \tag{5}$$

We see that $\theta(\phi)$ oscillates with a linear envelope as ϕ increases (*i.e.* the planet tilts out of the solar ecliptic). Also, I checked that at order $\mathcal{O}(c^2)$ there is a ϕ^2 envelope for oscillations that dominates at large ϕ, but given how small c will turn out to be and how young the solar system is, this modification is negligible.

In gr-qc/0412086, they bound motion of the planet Mercury. A year on Mercury lasts 90 earth days. If the solar system is 4 billions years old, then ϕ today is

$$\phi_{max} = \frac{365 \text{ days/year}}{90 \text{ days/Mercury year}}(4\times 10^9 \text{ years})(2\pi \text{ radians/Mercury year}) = 10^{11} \text{ radians}. \tag{6}$$

Insisting that $\Delta\theta(\phi_{max})$ is no larger than $7^\circ \sim .02$ radians, we can bound c (assume $\sin 2\theta_v \sim 1$):

$$c < \frac{.02\times 4}{\phi_{max}} \sim 10^{-12}. \tag{7}$$

Taking $v \sim 10^{-3}$, this gives a bound on α_2^{PPN} of

$$\alpha_2^{PPN} < 10^{-6}, \tag{8}$$

which is just a bit weaker than the bound we've been using.

Now, if we take $\ddot{\theta}(t=0)$ and just integrate to $T = 4$ billion years (as I suspect gr-qc/0412086 is doing), we would find

$$\ddot{\theta}(t=0) = \frac{c}{2}\sin 2\theta_v \frac{GM_\odot}{r_0^3}, \qquad \Delta\theta \sim c\sin 2\theta_v \frac{GM_\odot}{r_0^3}T^2 \sim c\sin 2\theta_v \phi_{\max}^2 \tag{9}$$

Insisting that the total θ transversed by Mercury is less than 7°, we would erroneously place a stronger bound on α_2^{PPN}:

$$c < \frac{.02}{\phi_{\max}^2} \sim 10^{-24}, \qquad \alpha_2^{PPN} \sim 10^{-18}, \tag{10}$$

which is within a factor of 10^{-3} of the bound on the bizarre combination of c_i's quoted in gr-qc/0412086.

Of course, this analysis does not explain why their effect goes as v, or why their result depends on the radius of the Sun, but at least I am convinced that if we apply the same type of bound on our model, the existing bound on α_2^{PPN} is stronger than the Mercury movement bound.