Correction to Universal Dynamics

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We had the expression for the spin-spin potential in ghost condensation for source of size R:

$$V(r) = \frac{M^2}{F^2}(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla}) \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{(M\vec{k}\cdot\vec{v}+i\epsilon)^2 - k^4} \frac{3(\sin kR - kR\cos kR)}{(kR)^3}. \tag{1}$$

We can write this as

$$V(r) = \frac{M^2}{8\pi F^2}(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla}) \left(r f(\alpha, \hat{r}\cdot\hat{v}, \gamma)\right) \tag{2}$$

where $\alpha = Mrv$, $\gamma = MRv$, $\cos\theta_v = \hat{r}\cdot\hat{v}$

$$f(\alpha, \hat{r}\cdot\hat{v}, \gamma) = \frac{8\pi}{\alpha} \int \frac{d^3\kappa}{(2\pi)^3} \frac{e^{i\vec{\kappa}\cdot\vec{\alpha}}}{(\vec{\kappa}\cdot\hat{v}+i\epsilon)^2 - \kappa^4} \frac{3(\sin\kappa\gamma - \kappa\gamma\cos\kappa\gamma)}{(\kappa\gamma)^3} \tag{3}$$

If we define

$$\vec{A}\diamond\vec{B} = \vec{A}\cdot\vec{B} - (\vec{A}\cdot\hat{r})(\vec{B}\cdot\hat{r}), \tag{4}$$

$$G_{ij} = \alpha^i f^{(i,j,0)}(\alpha, \hat{r}\cdot\hat{v}, \gamma) = \alpha^i \partial_\alpha^i \partial_{\hat{r}\cdot\hat{v}}^j f(\alpha, \hat{r}\cdot\hat{v}, \gamma), \tag{5}$$

then the potential is:

$$V(r) = \frac{M^2}{F^2}\frac{1}{8\pi r}\Big((G_{00}+G_{10}-G_{01}\hat{v}\cdot\hat{r})(\vec{S}_1\diamond\vec{S}_2) + (2G_{10}+G_{20})(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r}) \tag{6}$$

$$+ G_{02}(\vec{S}_1\diamond\hat{v})(\vec{S}_2\diamond\hat{v}) + G_{11}\left((\vec{S}_1\diamond\hat{v})(\vec{S}_2\cdot\hat{r}) + (\vec{S}_1\cdot\hat{r})(\vec{S}_2\diamond\hat{v})\right)\Big) \tag{7}$$

For $\gamma = 0$, $\theta_v = 0$:

$$G_{00}+G_{10} = \begin{cases} 0 & \alpha < 0 \\ 2\sin\alpha/\alpha & \alpha > 0 \end{cases} \tag{8}$$

$$G_{01} = \begin{cases} 1 & \alpha < 0 \\ \sin\alpha/\alpha & \alpha > 0 \end{cases} \tag{9}$$

$$2G_{10}+G_{20} = \begin{cases} 0 & \alpha < 0 \\ 2\cos\alpha - 2\sin\alpha/\alpha & \alpha > 0 \end{cases} \tag{10}$$

G_{02} and G_{11} are irrelevant, because when $\hat{v}\cdot\hat{r} = 1$, $\vec{S}\diamond\hat{v} = 0$.

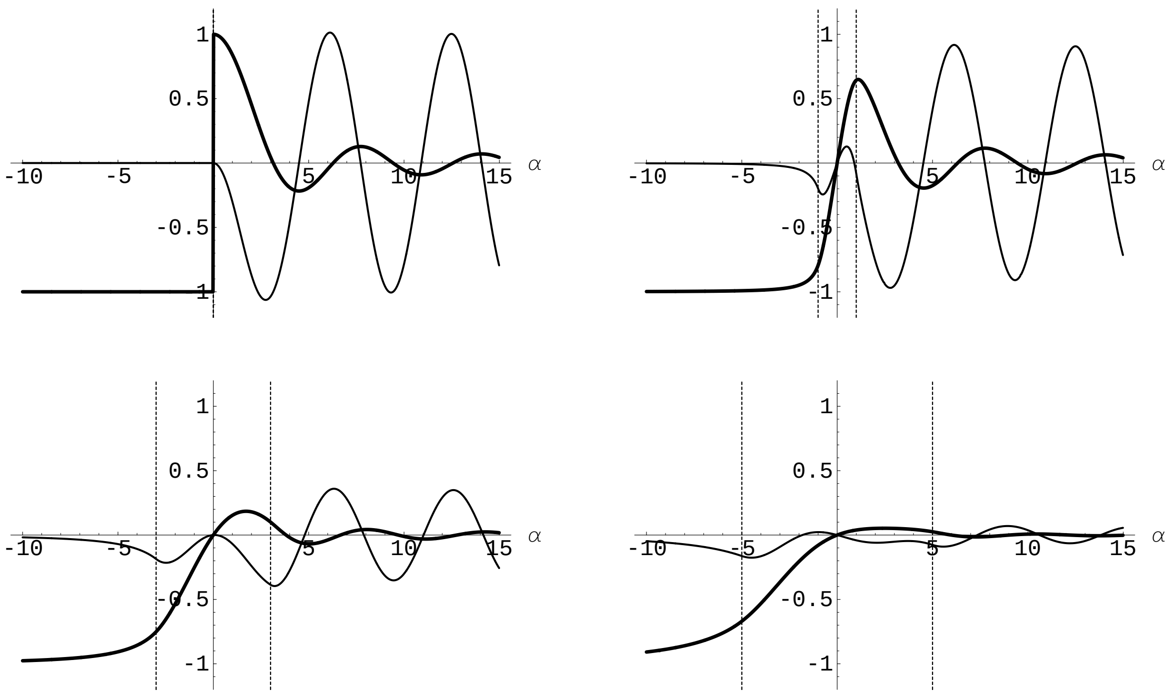

Figure 1: $\gamma = MRv = 0, 1, 3, 5$, $\theta_v = 0$. Bold: $G_{00} + G_{10} - G_{01}\hat{v} \cdot \hat{r}$, Light: $G_{10} + G_{20}/2$

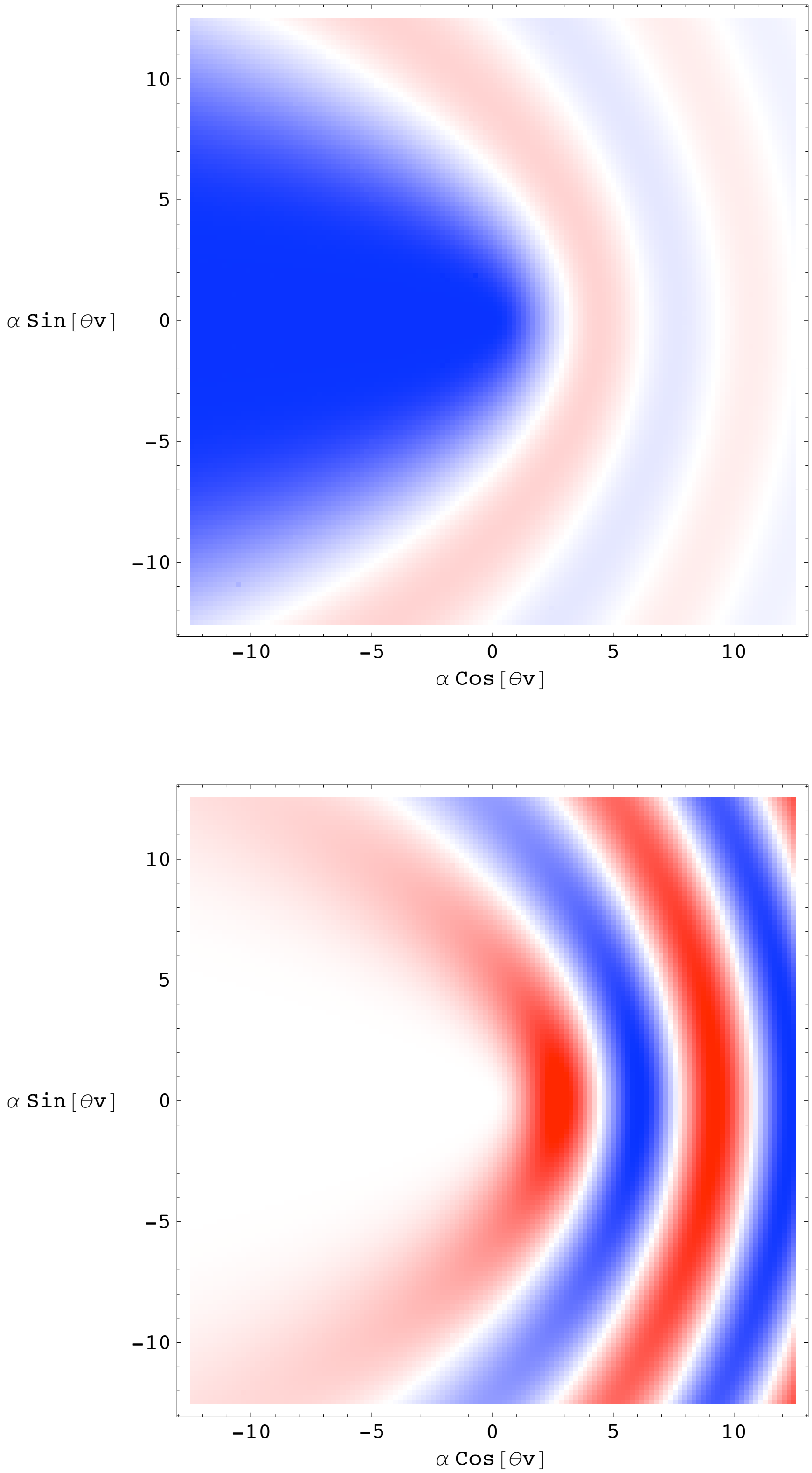

Figure 2: Up: $G_{00} + G_{10} - G_{01}\hat{v}\cdot\hat{r}$, Down: $G_{10} + G_{20}/2$

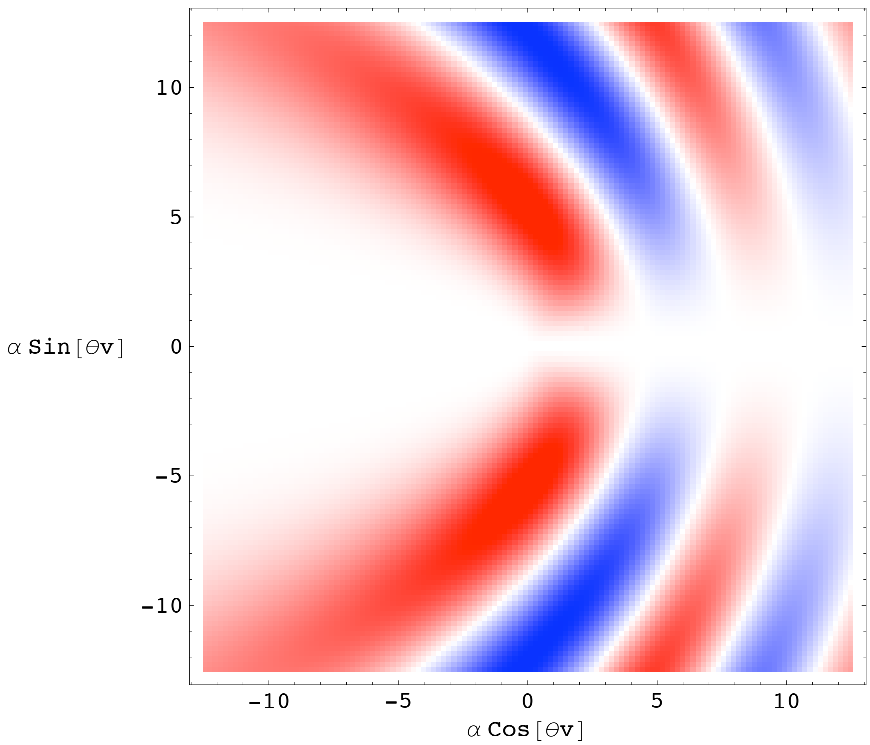

Figure 3: $G_{11} \sin^2 \theta_v$