Hi Kaustubh, Roberto and Alex,

Here are some comments about your email. We hope you find them helpful.

You correctly point out that in your paper, you obtain a hierarchy between v and f_π by a 10% cancellation in the top sector. We claimed that the tuning was between gauge and fermion contributions, and this is clearly false. We will correct that in our paper.

As for the Higgs potential in our model, the gauge contribution is parametrically of the form:

$$V(h)_{\text{gauge}} \sim \frac{g_{EW}^2 f_\pi^4}{N} \sin^4 \frac{h}{f_\pi} + \cdots, \tag{1}$$

where we are assuming that $m_\rho \sim 4\pi f/\sqrt{N}$. In order for this quartic to be large as in the original littlest Higgs, we want to have a small N theory, which is why we take large g_ρ. In addition, unless we have a large g_ρ, we cannot have a large enough W' mass unless we shrink the conformal window.

In the fermion sector, however, the potential is parametrically of the form:

$$V(h)_{\text{fermion}} \sim \frac{N_c \lambda_{\text{top}}^2 m_{t'}^2 f_\pi^2}{16\pi^2} \left(-\sin^2 \frac{h}{f_\pi} + \sin^4 \frac{h}{f_\pi} \right) + \cdots, \tag{2}$$

where $m_{t'}^2$ is the mass of the t' partner that cuts off the fermion contribution to the Higgs potential ($m_{t'} \sim \lambda_{\text{top}} f_\pi$). Note that this expression is *independent* of N. In your model, you say that the Higgs quartic from the fermion sector is parametrically $\lambda_h \sim N_c \lambda_{\text{top}}^2/(16\pi^2 N)$, whereas in our model:

$$\lambda_h \sim \frac{N_c \lambda_{\text{top}}^4}{16\pi^2}, \tag{3}$$

which again, has nothing to do with N because the fermion integrals are cut off way below m_ρ. Of course, for $N \sim \lambda_{\text{top}} \sim 1$, the quartics in both our models are numerically very similar.

For similar reasons, we do not expect two-loop calculations with large g_ρ to be particularly important in our model because of the largish separation between m_t' (and m_W') and m_ρ. In other words, $m_\rho \sim g_\rho f_\pi$ provides an effective cutoff for the low energy theory, but because much of the Higgs potential is already softened by the appearance of the t' and W' states, we expect the low energy perturbative estimate to be valid with $\Lambda \sim m_\rho$, even when g_ρ is at the edge of the (formally) perturbative regime.

Going back to the potential, to the extent that g_{EW} is large compared to the other couplings, our Higgs vev is approximately

$$v \sim \frac{\sqrt{N N_c} \lambda_{\text{top}}^2}{4\pi g_{EW}} f_\pi. \tag{4}$$

Numerically, this is not a huge separation (only a factor of 1/3), which is why we need to tune the Higgs doublet mass in our model. But parametrically, there is a large separation between v and f_π to the extent that λ_{top} is small. If we take $N \sim N_c \sim 1$, and all couplings of order g, then

$$v \sim \frac{g}{4\pi} f_\pi, \tag{5}$$

and no fine-tuning is required for this relation to hold. Of course, N and N_c are certainly not 1, and $\lambda_{\rm top} > g_{EW}$, but ignoring these numeric issues, we generically expect v to be a loop factor smaller than f_π. In your model, your potential is of the form:

$$V(h) \sim \alpha \cos \frac{h}{f_\pi} - \beta \sin^2 \frac{h}{f_\pi}, \tag{6}$$

so generically $v \sim f_\pi$, and there is no parametric reason why there should be a separation of scales. With 10% tuning you can enforce a separation of scales, but unlike the littlest Higgs where the small $\lambda_{\rm top}$ limit gives a very healthy composite Higgs theory with no tuning, there is no parametric limit of your theory where a natural separation is generated.

In effect, the $\sin^4 h/f_\pi$ piece to the potential which you need to alleviate fine-tuning is automatically provided by the little Higgs structure in the gauge sector (eq. 1). Indeed, the *raison d'être* for the littlest Higgs is to provide a naturally large quartic. Unfortunately, the top Yukawa is large, which complicates the Higgs potential from a numerical point of view. But to the extent that we trust parametrics—who would have thought that a 500 MeV kaon could be described by chiral perturbation theory?—we believe that the little Higgs structure gives a compelling explanation for a large separation between v and f_π.

Finally, we will reword our paper a bit to emphasize that your Higgs boson is within the LEP bounds. We welcome any further comments you might have.

Sincerely,
Jesse Thaler and Itay Yavin