A(QCd)S/(CCWZ)FT HLS/LH
or Little Technicolor

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Abstract

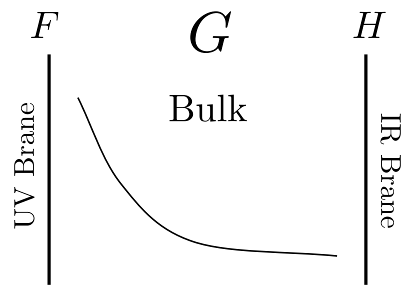
Inspired by the AdS/CFT correspondence, we show that any G/H symmetry breaking pattern can be described by a simple two-site moose diagram. While this construction is in principle already known in the context of Hidden Local Symmetry (HLS), we interpret this moose in a novel way to show that many little Higgs (LH) theories are actually contained in QCD. The G/H moose also illuminates the CCWZ prescription and provides a pedagogical way to understand phenomenological lagrangians. EXPAND?

1 Introduction

Whether or not strong dynamics ultimately explains the hierarchy problem, theories that exhibit confinement are undoubtedly the most elegant way to generate large hierarchies of scales [1, 2]. In the context of the standard model, there is the unfortunate reality that the electroweak scale (246 GeV), the precision electroweak scale ($\sim$ 10 TeV), and the flavor scale ($\sim$ 1000 TeV) are well separated, and no known strong dynamics can elegantly explain *three* hierarchies. In this light, little Higgs theories [3, 4, 5, 6, 7, 8, 9, 10] are a reasonable compromise to the hierarchy problem: strong dynamics are invoked to separate the precision electroweak scale from the Planck scale, collective breaking explains the "little hierarchy" [11, 12] between the confinement scale and the electroweak scale, and the flavor problem is presumably addressed by some sort of GIM mechanism [13].

For all the phenomenological successes of little Higgs theories, their structure can seem a bit artificial from the high energy perspective. While the $SU(5)/SO(5)$ littlest Higgs [5] could in principle arise from strong $SO(N)$ dynamics [14], the $(SU(3)/SU(2))^2$ simple group little Higgs [9] has no obvious QCD-like UV completion, because it is hard to imagine that an ordinary confining theory would break an $SU(N)$ flavor symmetry to $SU(N-k)$. In this light, technicolor-like theories [15, 16] seem a lot more realistic (if not phenomenologically viable) in that they are simply scaled-up versions of ordinary $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_D$ chiral symmetry breaking in QCD. But before we dismiss the little Higgs theories as interesting but artificial constructions, we will actually show that *any* $(SU(N)/H)^n$ little Higgs theory can arise from n copies of QCD with N flavors! In other words, the little Higgs could literally be a pion of QCD.

The starting point for our analysis is the AdS/CFT correspondence [17, 18, 19] and its phenomenological interpretation [20, 21]. There is a straightforward way to construct the AdS dual of a CFT that yields a G/H nonlinear sigma model at low energies and where a subgroup $F \subset G$ is gauged: simply consider a slice of AdS_5 [22] with bulk G gauge bosons where the gauge symmetry is reduced to F on the UV brane and H on the IR brane [23]:



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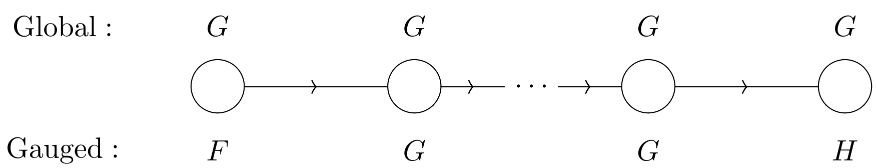
This construction was studied in the context of the littlest Higgs in [24]. In this paper, we take the obvious (in retrospect) next step and deconstruct the warped dimension [25]. The link fields in the moose are the Wilson lines constructed out of A_5, and the warp factor is reflected in the different decay constants on the links [26].



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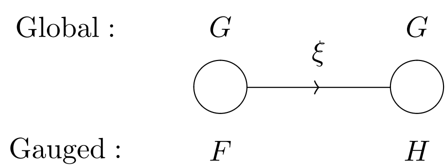
Going the extreme where we only introduce sites corresponding the UV and IR branes, we arrive at a moose diagram which at low energies is supposed to describe a G/H non-linear sigma model with $F \subset G$ gauged.



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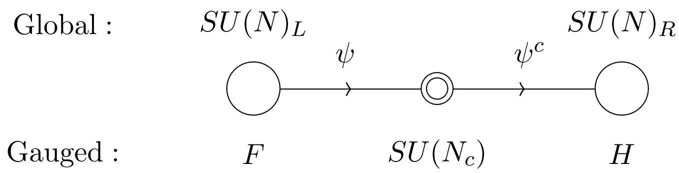
In this AdS/CFT-inspired moose, the subgroup H (which was a global symmetry from the low energy perspective) has become a gauge symmetry! Furthermore, we will see that the link field ξ has a nice interpretation vis à vis the CCWZ prescription [27, 28].

Now, this construction is actually quite well-known and is referred to in the literature as Hidden Local Symmetry (HLS) [29, 30, 31]. Normally, the HLS construction is used as a model for the ρ mesons in confining theories, and in the AdS picture, the H gauge symmetry is indeed the "gauge symmetry" associated with light spin-1 KK modes which are holographically identified as CFT resonances. This moose was implicitly studied in [32] to try to understand the UV sensitive parameters in the $SU(5)/SO(5)$ littlest Higgs, and they found that by including a $SO(5)$s worth of ρ mesons, all the UV sensitive parameters in the original littlest Higgs became finite to one-loop order. So to date, HLS has been a way to discuss the spin-1 phenomenology of confining theories below Λ_{QCD}. In this paper, we make the not-so-radical observation that the H gauge symmetry could survive deep into the ultraviolet!

In the case that $G = SU(N)$, we can write down a simple UV completion for the moose diagram in eq. (3).



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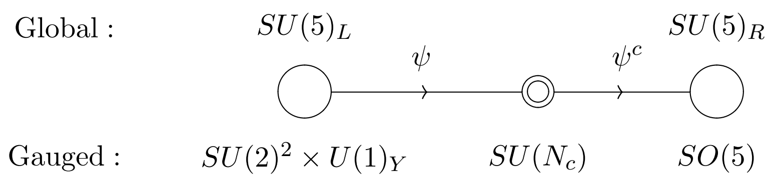
When $SU(N_c)$ confines, the fermion condensate $\langle \psi \psi^c \rangle$ will become the link field ξ. Specializing to the littlest Higgs (with only one hypercharge generator):



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Apparently, the $SU(5)/SO(5)$ littlest Higgs is contained in ordinary QCD with five flavors, where some of the flavor symmetries have been gauged. In other words, this theory *is* technicolor, albeit with different gauge groups.

As we will see, this remarkably prosaic construction has exactly the same low energy phenomenology as the original littlest Higgs, despite the fact that we have introduced a new gauge symmetry. Of course, now that we understand that little Higgs theories could come from technicolor-like theories, we inevitability encounter some of the same tensions from technicolor, such as how to generate fermion Yukawa couplings without introducing new sources of flavor violation [33, 34]. On the other hand, given how difficult it was to even imagine a UV completion for little Higgs theories, it is absolutely remarkable that such a simple construction even exists.

In the next section, we review the AdS/CFT inspiration for the G/H moose, and show how there is a corresponding story for any extra dimension with boundaries. In Section 3, we show how the G/H moose illuminates and simplifies the CCWZ prescription by turning any G/H phenomenological lagrangian into an exercise in chiral symmetry breaking. We return briefly to AdS space in Section 4 to understand boundary condition breaking more transparently. In Section 5, we examine the $(SU(3)/SU(2))^2$ simple group little Higgs to show how the same low energy degrees of freedom can arise from four very different theories: a straightforward application of AdS/CFT, two copies of QCD, one copy of QCD in the vector limit [35], and a known AdS_5 construction [23] that looks nothing like a little Higgs theory (but secretly is). Using the $SU(5)/SO(5)$ littlest Higgs, we show in Section 6 some of the tensions in trying to incorporate fermions in this construction. We conclude with some outstanding questions about more general little Higgs theories and speculations on the Wess-Zumino-Witten term [36, 37].

2 Warped Dimensions and Nonlinear Sigma Models

In this section, we review how to identify the A_5 component of a bulk gauge field with a nonlinear sigma model link field ξ. We derive the effective lagrangian for ξ in any theory where the extra dimension is an interval and the 4D slices are Minkowski space. In the next section, we show how this lagrangian dovetails nicely into the well-known CCWZ prescription for nonlinear realizations of broken symmetries.

We first note that this construction is unique to extra dimensions with boundaries. If we had a compactified flat extra dimension with circumference R and bulk gauge fields, there would be two massless states: a spin-1 zero mode ($A_\mu^{(0)}$) and spin-0 zero mode ($A_5^{(0)}$). However, with the minimal gauge kinetic lagrangian, there is zero amplitude for $A_5^{(0)} A_5^{(0)}$ scattering, so the effective pion decay constant for $A_5^{(0)}$ is $f_\pi = \infty$. We can see this in two ways. First, by KK momentum conservation, there is no

$$A_5^{(0)} A_5^{(0)} \partial^\mu A_\mu^{(n)} \tag{6}$$

vertex, so when we integrate out the heavy gauge modes, we do not generate a

$$(A_5^{(0)} \partial_\mu A_5^{(0)})^2 \tag{7}$$

four-point interaction. Second, we can imagine deconstructing the compactified dimension with N link fields. Each link U_i has decay constant f, and the product of the link fields $U = U_1 U_2 \cdots U_N$ plays the role of the A_5 zero mode. It is easy to show that the effective decay constant f_π for U is $f_\pi^2 = N f^2$. In terms of the 5D gauge coupling g_5 and the lattice spacing $a = R/N$,

$$f^2 = \frac{1}{g_5^2 a}, \qquad f_\pi^2 = N f^2 = \frac{N}{g_5^2 a} = \frac{1}{g_4^2 a^2}, \tag{8}$$

where $g_4 = g_5/\sqrt{R}$ is the effective 4D gauge coupling. For fixed lattice spacing, f_π is finite, but as we go to the continuum theory $a \to 0$, the pion decay constant f_π goes to infinity. In other words, with a homogeneous compact extra dimension, the A_5 zero mode is simply a free scalar field, and cannot be interpreted as a nonlinear sigma model.

When the extra dimension is an interval with fixed boundaries (or a compact space with defects [3]), there is no longer KK momentum conservation, and we can generate an effective $A_5^{(0)}$ four point interaction. In flat space, we can easily estimate f_π. The coefficient of the vertex in eq. (6) is g_4 because it arises from gauge interactions. For an interval of length R, the mass of the first KK mode is $1/R$, so integrating out the massive mode, the vertex in eq. (7) has coefficient $g_4^2 R^2$. In terms of the pion decay constant,

$$\frac{1}{f_\pi^2} (A_5^{(0)} \partial_\mu A_5^{(0)})^2, \qquad f_\pi = \frac{1}{g_4 R}. \tag{9}$$

As expected, f_π is proportional to the only mass scale in the theory $1/R$, and goes to infinity as the gauge coupling goes to zero.

Of course, this argument assumed that such an $A_5^{(0)}$ zero mode existed, and this depends on the boundary conditions on the interval. (We discuss boundary conditions more carefully in Section 4.) If A_μ has Neumann boundary conditions at either boundary, then we can simply go to $A_5 = 0$ gauge, and there is no candidate for the nonlinear sigma field. This is also obvious from the AdS/CFT correspondence. To describe a G/H nonlinear sigma model with $F \subset G$ gauged, we construct a slice of AdS_5 with bulk G gauge bosons where Dirichlet boundary conditions reduce the gauge symmetry to F on the UV brane and to H on the IR brane [23]. If A_μ has Neumann boundary conditions on the UV brane, then $F = G$ so any would-be Goldstones are eaten by the Higgsing of G to H. Similarly, with Neumann boundary conditions on the IR brane, then $H = G$ and there is no spontaneous symmetry breaking in the dual theory. The only way that there can be uneaten (psuedo-)Goldstone bosons is if some components of A_μ have Dirichlet boundary condition on both branes, and the A_5 zero modes in $G/(F \cup H)$ are indeed the uneaten Goldstones.

In an arbitrary warped space, we can figure out the pion decay constant for the Goldstones most simply by deconstructing the fifth dimension. Consider a metric for an extra dimensional interval with 4D Minkowski slices:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{10}$$

where $0 < y < R$, and $\sigma(y)$ is an arbitrary function. Assuming no boundary kinetic terms, the action for bulk G gauge bosons is:

$$S = -\frac{1}{2}\int d^4x\, dy\, \frac{1}{g_5^2}\,\mathrm{tr}\left(F_{\mu\nu}^2 - 2e^{-2\sigma}F_{\mu 5}^2\right). \tag{11}$$

We can deconstruct this action with $N+1$ sites, N links, and equal lattice spacing a [26]:

$$A_\mu^0 \xrightarrow{U_1} A_\mu^1 \xrightarrow{U_2} \cdots \xrightarrow{U_{N-1}} A_\mu^{N-1} \xrightarrow{U_N} A_\mu^N \tag{12}$$

The action for this system is

$$S = \int d^4x \left(-\frac{1}{2}\sum_{i=0}^{N}\frac{a}{g_5^2}\,\mathrm{tr}(F_{\mu\nu}^i)^2 + \sum_{i=1}^{N}\frac{e^{-2\sigma_i}}{g_5^2 a}\,\mathrm{tr}\,|D_\mu U_i|^2\right), \tag{13}$$

where $\sigma_i = \sigma(ai)$, and the link field covariant derivative is

$$D_\mu U_i = \partial_\mu U_i + iA_\mu^{i-1}U_i - iU_i A_\mu^i. \tag{14}$$

At this point, we have not specified the boundary conditions on the end sites. We take $A_\mu^0 = 0$, which corresponds to not gauging any subgroup of the G global symmetry in the dual theory. (We will consider the case $A_\mu^0 \in F$ in the next section.) Similarly, we take $A_\mu^N \in H$ to account for the spontaneous breaking of G to H in the dual theory. We can go to a unitary gauge where $U_i = 1$ for $1 \leq i \leq N-1$ and $U_N \equiv \xi \in G/H$. The spectrum consists of the massless Goldstones in ξ, and a tower of massive spin-1 fields.

It is straightforward to integrate out these heavy spin-1 fields to arrive at an effective lagrangian for ξ. Going to canonical normalization for the gauge fields and ignoring the gauge kinetic terms, eq. (13) becomes with our gauge choice:

$$\mathcal{L} = m_1^2\,\mathrm{tr}(A_\mu^1)^2 + \sum_{i=2}^{N-1} m_i^2\,\mathrm{tr}\left(A_\mu^{i-1} - A_\mu^i\right)^2 + m_N^2\,\mathrm{tr}\left|\frac{\sqrt{a}}{g_5}\partial_\mu\xi + iA_\mu^{N-1}\xi - i\xi A_\mu^N\right|^2, \tag{15}$$

where $m_i = e^{-\sigma_i}/a$. Integrating out the massive A_i fields:

$$\mathcal{L} = f_{\text{eff}}^2\,\mathrm{tr}\,|\xi^\dagger\partial_\mu\xi|^2 + 2f_{\text{eff}}^2\sum_a\left(\mathrm{tr}\,T^a\xi^\dagger\partial_\mu\xi\right)^2, \tag{16}$$

where T_a are the generators of H, and

$$\frac{1}{f_{\text{eff}}^2} = g_5^2 a \sum_{i=1}^{N} e^{2\sigma_i}. \tag{17}$$

More compactly, we can decompose $\xi^\dagger \partial_\mu \xi$ into elements of H and G/H:

$$\xi^\dagger \partial_\mu \xi \equiv v_\mu + p_\mu, \qquad v_\mu \in H, \qquad p_\mu \in G/H. \tag{18}$$

This allows us to rewrite eq. (16) as

$$\mathcal{L} = f_{\text{eff}}^2 \, \mathrm{tr} \, p^\mu p_\mu^\dagger. \tag{19}$$

At we will discuss in the next section, this is precisely the CCWZ kinetic term for a G/H nonlinear sigma field. In other words, when we integrate out the bulk of the interval extra dimension, the low energy degrees of freedom are just the Goldstones, as had to be the case from the AdS/CFT correspondence.

Going to the continuum limit of eq. (17), the pion decay constant for ξ is

$$\frac{1}{f_{\text{eff}}^2} = g_4^2 R \int dy \, e^{2\sigma(y)}. \tag{20}$$

In the case of a flat extra dimension, $f_{\text{eff}} = 1/g_4 R$ as we guessed in eq. (9). In AdS space, the warp function is $\sigma(y) = ky$, and in the limit $kR \gg 1$,

$$f_{\text{eff}} = \frac{e^{-kR} k \sqrt{2}}{g_4 \sqrt{kR}}. \tag{21}$$

This result is parametrically what we expect from AdS/CFT. It is customary to define the AdS perturbative expansion parameter $g_\rho = g_4 \sqrt{kR}$, which can be holographically identified as $g_\rho \sim 4\pi/\sqrt{N}$ in a large N CFT [20]. The scale ke^{-kR} can be roughly identified with the confinement scale Λ_{QCD}. So we see that in AdS space

$$f_{\text{eff}} \sim \frac{\Lambda_{QCD}}{4\pi} \sqrt{N}, \tag{22}$$

giving the expected $\sqrt{N}$ scaling of f_{eff} from the dual CFT description [38].

While the AdS/CFT correspondence informed this construction of a G/H nonlinear sigma model from a warped dimension, from eq. (20) we see that we can generate a non-infinite pion decay constant from any extra dimension with boundaries, including flat space. One advantage of AdS space is that integrating out heavy modes is almost exactly the same as integrating out sites of the moose. If the lattice spacing a is much larger than $1/k$, then in eq. (15) we see that $m_1 \gg m_2$, so integrating out A_1 is almost exactly the same as integrating out the heaviest mode in the spectrum. In continuum language, we can move the UV brane while keeping the low energy physics fixed by allowing couplings on the UV brane to run, and this running is holographically dual to renormalization group running in the CFT. For the same reason, if we wanted to move the IR brane while keeping the low energy physics fixed, we would have to introduce new degrees of freedom on the IR brane in addition to allowing couplings to change. This takes into account the fact that by moving the IR brane, we are unintentionally integrating out a light degree of freedom which has $\mathcal{O}(1)$ overlap with A_μ^N, so we have to integrate this degree of freedom back into the spectrum. In flat space, integrating out heavy modes involves all of the sites, and therefore introduces interactions between the remaining sites that are non-local in theory space. In AdS space, the induced non-local interactions are exponentially suppressed and can be ignored to an excellent approximation.

3 Pedagogical CCWZ

Now that we understand the AdS/CFT inspiration for how to construct a G/H nonlinear sigma model from an extra dimension, we can dispense with the 5D picture completely by integrating out the bulk and reducing the theory to a two-site moose diagram. In this section, we will see how this trivializes the CCWZ construction of phenomenological lagrangians.

When a global symmetry G spontaneously breaks to H, there is a G/H's worth of Goldstone bosons which transform nonlinearly under G. At energies beneath the symmetry breaking scale, the low energy fields only have well-defined transformation properties under H. The CCWZ prescription [27, 28] tells us how to write down a phenomenological lagrangian for the Goldstones and the low energy fields which is manifestly H invariant.

While the CCWZ prescription is well-defined, the construction of the low energy effective lagrangian for a general G/H can appear somewhat mysterious (or alternatively, divinely inspired). If T^a are the generators of H, and X^a are the generators of G/H, we introduce a field ξ

$$\xi = e^{i\pi^a X^a/f_\pi} \tag{23}$$

that transforms as

$$\xi \to g\xi h^\dagger, \tag{24}$$

where g is an element of G, and $h \in H$ is a function of π^a that furnishes a non-linear representation of G. The object ξ interpolates between fields that transform linearly under H to fields that transform linearly under G. In the case that G/H is a symmetric space, it is easy to write down kinetic terms for ξ. A symmetric space has a $\mathbf{Z}_2$ automorphism under which:

$$T^a \to T^a, \qquad X^a \to -X^a. \tag{25}$$

We can then construct the object $\Sigma = \xi\tilde{\xi}^\dagger = \xi^2$, where tildes indicate the image of a field under eq. (25). Σ has the nice property that it transforms linearly under G:

$$\Sigma \to g\Sigma\tilde{g}^\dagger. \tag{26}$$

Therefore, the kinetic term for Σ is simply

$$\mathcal{L}_\Sigma = \frac{f^2}{4}\,\mathrm{tr}\,|D_\mu\Sigma|^2, \qquad D_\mu\Sigma = \partial_\mu\Sigma + iF_\mu\Sigma - i\Sigma\tilde{F}_\mu, \tag{27}$$

where F_μ is the gauge field associated with an F subgroup of G.

For general G and H, however, the Goldstone kinetic terms are not particularly obvious. If we gauge a subgroup F of G, we can decompose the object $\xi^\dagger D_\mu\xi$ into elements of G and G/H:

$$\xi^\dagger(\partial_\mu + iF_\mu)\xi \equiv v_\mu^a T^a + p_\mu^a X^a. \tag{28}$$

Under H, the objects $v_\mu = v_\mu^a T^a$ and $p_\mu = p_\mu^a X^a$ have nice transformation properties:

$$v_\mu \rightarrow h(v_\mu + \partial_\mu)h^\dagger, \tag{29}$$

$$p_\mu \rightarrow h p_\mu h^\dagger. \tag{30}$$

The field v_μ transforms like a connection and can be used to write down G invariant kinetic terms for fields that transform linearly under H. For example, the kinetic term for a fermion ψ that transforms as $\psi \rightarrow h\psi$ is:

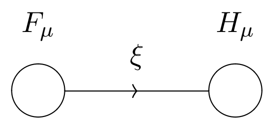
$$\mathcal{L}_\psi = \bar{\psi} i \bar{\sigma}^\mu (\partial_\mu + v_\mu)\psi. \tag{31}$$

We can use p_μ to write down kinetic terms for the Goldstones themselves:

$$\mathcal{L}_{\text{kinetic}} = f^2 \operatorname{tr} p^\mu p_\mu^\dagger, \tag{32}$$

but the form of p_μ depends heavily on the specific groups F, G and H. The magic of the CCWZ prescription is to realize that the low energy lagrangian should depend on ξ through the fields v_μ and p_μ.

From the point of view of the G/H moose, however, it is completely trivial to understand why v_μ and p_μ show up at all. The moose diagram from eq. (3) is



(33)

where $F_\mu \in F$, $H_\mu \in H$, and ξ transforms as a bifundamental under the approximate G global symmetries on each of the sites. The effective lagrangian is

$$\mathcal{L} = -\frac{1}{2g_F^2} \operatorname{tr} F_{\mu\nu}^2 - \frac{1}{2g_H^2} \operatorname{tr} H_{\mu\nu}^2 + f^2 \operatorname{tr} |D_\mu \xi|^2, \qquad D_\mu \xi = \partial_\mu \xi + i F_\mu \xi - i \xi H_\mu. \tag{34}$$

We can go to a unitary gauge where $\xi \in G/(F \cup H)$. At tree level, the spectrum consists of the massless Goldstones from ξ, the massless gauge bosons in $F \cap H$, and the massive gauge bosons in $F \cup H$. The gauge coupling for the massless gauge bosons is

$$\frac{1}{g^2} = \frac{1}{g_F^2} + \frac{1}{g_H^2}. \tag{35}$$

The mass of the massive gauge bosons are:

Generators	Mass
$F \setminus H$	$g_F f$
$F \cap H$	$\sqrt{g_F^2 + g_H^2} f$
$H \setminus F$	$g_H f$

(36)

In the limit that $g_H \rightarrow \infty$, the massive gauge bosons in H decouple, and we can simply integrate out H_μ. To leading order, the H_μ equation of motion sets

$$i H_\mu \equiv v_\mu, \tag{37}$$

where v_μ is defined by eq. (28). Plugging this value of H_μ into eq. (34) in the $g_H \to \infty$ limit:

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{tr}\,F_{\mu\nu}^2 + f^2\,\mathrm{tr}\,p^\mu p_\mu^\dagger, \tag{38}$$

in agreement with the CCWZ expectation. Similarly, if a fermion ψ is charged under H, $\psi \to h\psi$, its kinetic term before and after integrating out H_μ is

$$\mathcal{L}_\psi = \bar{\psi} i\bar{\sigma}^\mu(\partial_\mu + iH_\mu)\psi, \qquad \mathcal{L}_\psi = \bar{\psi} i\bar{\sigma}^\mu(\partial_\mu + v_\mu)\psi + \frac{1}{f^2}\,(\text{four-fermion operators})\,, \tag{39}$$

in agreement with CCWZ and naïve dimensional analysis [39, 40] for the size of four fermion operators. More generally, for finite g_H, it is easy to show that the size of higher order interactions from integrating out H_μ are consistent with NDA with the choice $\Lambda = g_H f$.

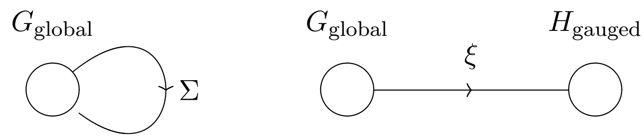
We see that integrating out the H_μ site from the G/H moose reduces the theory to an ordinary G/H non-linear sigma model. The equation of motion sets gauge connection field H_μ equal to the the CCWZ connection field v_μ, and using the H_μ equation of motion, the link field kinetic term for ξ becomes a function of p_μ only. Whereas the original CCWZ prescription for arbitrary G/H required some insight and forethought to understand the importance of v_μ and p_μ, the lagrangian in eq. (34) is just as trivial as the Goldstone lagrangian in eq. (27) for symmetric spaces, and by integrating out H_μ, the CCWZ prescription falls out immediately. In our opinion, this is the simplest way to understand the structure of arbitrary phenomenological lagrangians, especially if one is familiar with AdS/CFT.

4 Boundary Conditions vs. Linear Sigma Models

In Section 2, we used Dirichlet boundary conditions to reduce the gauge symmetry on the UV and IR branes. One might worry that this construction is not entirely general because it is also possible to reduce the symmetries by linear sigma models on the branes (or even by brane-localized technicolor). Boundary condition breaking is the limit that the vevs of the linear sigma fields go to infinity, and it may seem like we are neglecting an additional free parameter by always going to this limit.

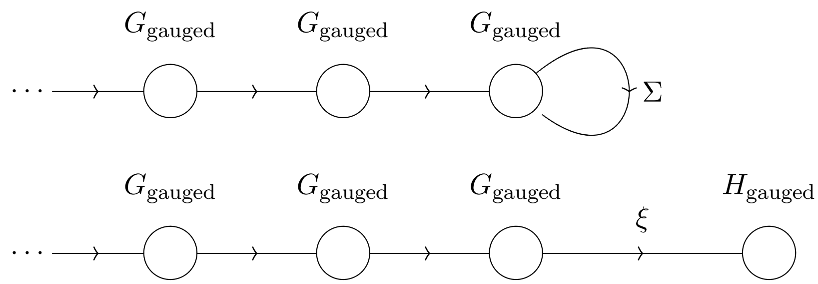
In the case that the vev is much smaller than the scale of the IR brane, then we might as well not even talk about AdS space, because the low energy degrees of freedom (including the radial modes of the linear sigma field) are well separated from the AdS KK excitations. In the case that the vev is comparable to or larger than the IR brane scale, then the details of how the symmetries are broken are irrelevant for low energy physics, so all we care about is the resulting non-linear sigma field on the IR brane after symmetry breaking.

It is easy to see that a non-linear sigma field on the IR brane is equivalent to a different AdS space with boundary condition breaking on the IR brane but with a slightly distorted warp factor. (The identical story holds for UV brane symmetry breaking.) Imagine a non-linear sigma field Σ for a symmetric space G/H. In the previous section, we showed that the following moose diagrams have identical low energy physics:



(40)

where $\Sigma = \xi^2$, and it is understood that Σ transforms as $\Sigma \to g\Sigma\tilde{g}^\dagger$. If we insert the non-linear sigma field on the IR brane, the G global symmetry is identified with the bulk G gauge symmetry. Deconstructing the AdS space, we have two theories with identical low energy physics:



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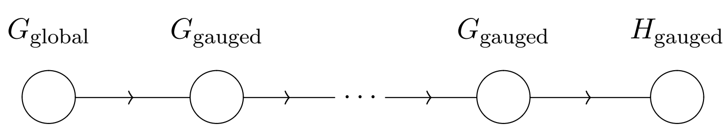
The first moose is just the deconstructed version of non-linear sigma field living on the IR brane. The second moose can now be interpreted as a deconstructed AdS space where the IR brane is slightly shifted and boundary conditions reduce the gauge symmetry to H on the IR brane. In order to account for the fact that the decay constant on the ξ link might not track the standard warp factor when we reconstruct the AdS space, we would have to allow the function $\sigma(y)$ to be slightly perturbed near the IR brane.

In fact, there is no reason why we couldn't put the N site moose



(42)

on the IR brane and identify $G_{\rm global}$ with the bulk gauge symmetry. As we saw in Section 2, this moose also describes a G/H non-linear sigma model at low energies. The point is that it hardly matters how one breaks $G \to H$ on the IR brane because all the relevant information for how it is broken can be absorbed into a redefinition of the warp factor near the IR brane. Similarly, it is often computationally simpler to work with an IR brane non-linear sigma field instead of boundary condition breaking, and from eq. (17), we see that the decay constant for Σ (which is the same as the decay constant for ξ) can be taking to infinity but the effective G/H pion decay constant will still be finite.

Another interesting question is what fields are responsible for unitarizing Goldstone-Goldstone scattering in AdS space. This is relevant both in Higgsless theories [41], where the Goldstones are the longitudinal modes of the W and Z bosons, and in composite Higgs models [23, 42] where some of the Goldstones comprise the Higgs boson. In the context of the littlest Higgs, there should be fields in the AdS construction of [24] that restore perturbative unitarity of Goldstone scattering [43]. If G is broken to H on the IR brane through a linear sigma model *and* the vev of the linear sigma field is much smaller than the IR brane scale, then unitarity is restored by the radial modes of the linear sigma field, in analogy with the ordinary Higgs mechanism.

When the linear sigma vev is comparable to the IR brane scale, however, the IR brane Goldstones mix with the bulk gauge fields, and it is no longer accurate to think of the physical Goldstones as being entirely localized on the IR brane. In that case, one expects that unitarity is restored by some combination of the radial modes of the linear sigma field and the KK gauge bosons that have the same quantum numbers as the radial modes. The radial modes fill an adjoint and a singlet of H, and there are certainly KK gauge bosons that transform as adjoint of H, namely those with Neumann boundary conditions on the IR brane. In the limit that the linear sigma vev goes to infinity, the KK gauge bosons with Dirichlet boundary conditions on the IR brane are singlets under H, and we also expect that there will be other singlet states corresponding to broad resonances of the CFT ("QCD string" states) [20]. The point which is made obvious by the two-site deconstruction is that even when the linear sigma vev goes to infinity, there is still a state in the theory that has the quantum numbers to potentially address Goldstone-scattering unitarity, namely the H_μ field. Whether unitarity is actually restored for arbitrary boundary conditions of course requires a more complete analysis [41].

5 Four Views of the Simple Group Little Higgs

While the H_μ field can be a model of the ρ-like mesons in HLS or as an auxiliary field in pedagogical CCWZ, the key point of this paper is that it is entirely consistent for the H_μ field to be a real gauge field that survives deep into the ultraviolet. The simple group little Higgs model [9] is an ideal laboratory to study this possibility, because the symmetry structure of the theory makes it possible to imagine many different UV completions. In essence, we will show that the distinction between W'-like gauge bosons and ρ-like mesons can be blurred in these models. Moreover, we will understand that holographic composite Higgs models are secretly little Higgs theories in the sense that there is a meaning to collective breaking.

Ignoring hypercharge for simplicity, the simple group little Higgs is based on an $(SU(3)/SU(2))^2$ symmetry breaking pattern where the diagonal $SU(3)$ symmetry is gauged. Using the AdS/CFT correspondence, we can easily construct an AdS$_5$ model to recover the low energy physics of the simple group theory. The brane and bulk gauge symmetries are:

	Gauge Symmetry
UV Brane	$SU(3)_V$
Bulk	$SU(3)_1 \times SU(3)_2$
IR Brane	$SU(2)_1 \times SU(2)_2$

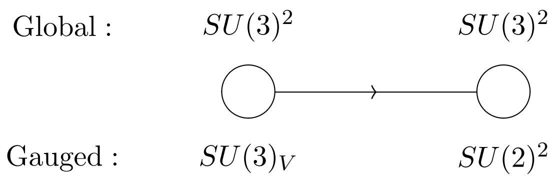
(43)

At low energies, there are the massless $SU(2)_{EW}$ gauge bosons and an $SU(3)/SU(2)$'s worth of Goldstones, which contains an electroweak doublet (the Higgs) and an electroweak singlet.

Like all little Higgs theories, the simple group theory exhibits collective breaking in that no single interaction breaks the global symmetries that protect the Higgs, so the Higgs mass is not quadratically sensitive to the cutoff (*i.e.* the confinement scale). It is easy to see how collective breaking works in AdS space in terms of boundary conditions on the UV and IR branes [24]. When the gauge boson boundary conditions are Neumann on either brane, then we can go to $A_5 = 0$

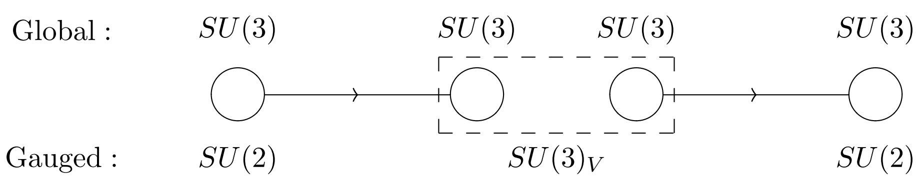
gauge, and all the Goldstones are exact (*i.e.* they are all eaten). Similarly, when the gauge boson boundary conditions are Dirichlet on either brane, then there are no massless gauge fields for the Goldstones to mix with, so all the Goldstones are exact. Only when different components of the bulk gauge field have different boundary conditions on each brane is it possible to have pseudo-Goldstone bosons. In the simple group theory, there are three interactions that have to be turned on for the symmetries that protect the Higgs to be broken: $SU(3)_V$ has to be gauged on the UV brane, $SU(3)_1$ has to be reduced to $SU(2)_1$ on the IR brane, and similarly for $SU(3)_2$.

The lightest KK modes with Neumann boundary conditions on the IR brane are holographically identified with the ρ-like mesons of the confining CFT. We will now perform the two-site deconstruction of the AdS_5 model in eq. (43), to show that these ρ mesons can be interpreted as W' gauge bosons in a theory of two copies of QCD with $N_f = 3$. The relevant moose diagram is:



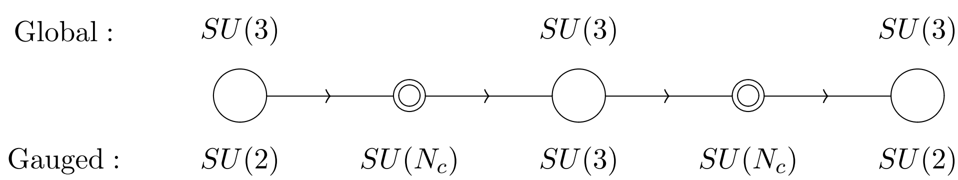
(44)

Because $SU(3)^2$ is a direct product space, we can decompose this moose into two pieces:



(45)

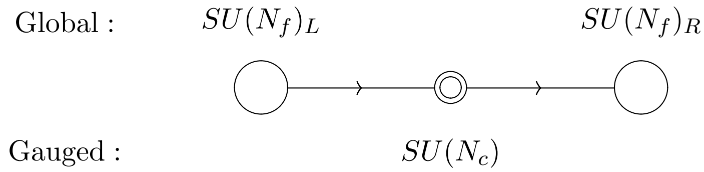
The $SU(3)_V$ gauge symmetry breaks the approximate $SU(3)$ global symmetries on the boxed sites, so we might as well combine those two-sites into a single site. We can now UV complete the link fields into fermion condensates from two $SU(N_c)$ confining theories (arrows now correspond to fermions):



(46)

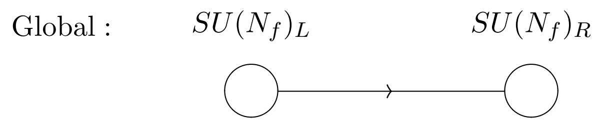
We see that the low energy degrees of freedom in the simple group theory can arise from two copies of QCD with $N_f = 3$, where some of the flavor symmetries have been gauged. Indeed, the $SU(2)^2$ gauge symmetries which had been associated with the holographic ρ-like mesons in the AdS_5 model are now ultraviolet gauge symmetries associated with W and W' gauge fields. If we had deconstructed the original AdS_5 model with three sites instead of two, we could do the exact same construction to see that the simple group theory could be embedded into four copies of QCD, each with $N_f = 3$ and the appropriate flavor symmetries gauged. It is obvious from this construction that any $(SU(N)/H)^n$ little Higgs theory where some subgroup of $SU(N)^n$ is gauged can arise from n copies of QCD with N flavors with the appropriate gauging of the flavor symmetries.

There is an even more interesting UV completion of the moose in eq. (45) inspired by the HLS model of the ρ meson in ordinary QCD [29]. As postulated by [35], there may be a "vector limit" of QCD (most likely the large N_c limit) where the ρ meson is anomalously light. Above the confinement scale, the QCD moose with N_f flavors is:



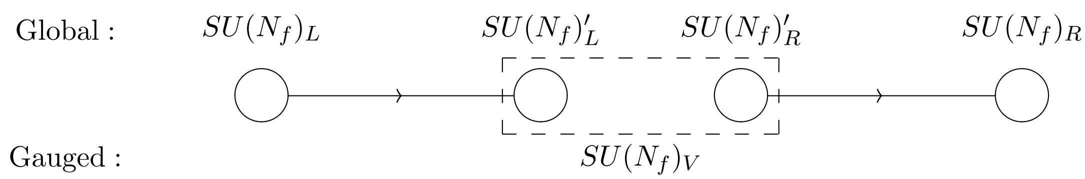
(47)

When we describe only the pions of QCD, the low energy description is in terms of the non-linear sigma model moose:



(48)

The hypothesis of [35] is that the effective description of the pions and ρ mesons in the vector limit of QCD should be:



(49)

where $SU(N_f)_V$ is the gauge symmetry associated with the ρ mesons. The point of the vector limit is that as the $SU(N_f)_V$ gauge coupling is taken to zero, there could be an enhanced $SU(N_f)^4$ "flavor" symmetry in QCD which would allow the longitudinal components of ρ to be the exact chiral partners of the pions.

In fact, the AdS$_5$ metaphor for large N_c QCD with N_f flavors (and no flavor symmetries gauged) realizes this scenario explicitly. The gauge symmetries in the AdS$_5$ metaphor are:

	Gauge Symmetry
UV Brane	$\emptyset$
Bulk	$SU(N_f)_L \times SU(N_f)_R$
IR Brane	$SU(N_f)_V$

(50)

and when we deconstruct the AdS space, we indeed generate the moose in eq. (49). What is amusing is that in ordinary QCD, $SU(N_f)_V$ is the gauge symmetry associated with the ρ meson, whereas in the original inspiration for the simple group little Higgs, $SU(3)_V$ is a gauge symmetry associated with an ultraviolet gauge boson. From the point of view of low energy degrees of freedom, however, the mooses in eqs. (45) and (49) are identical if we set $N_f = 3$ and gauge an $SU(2)^2$ subgroup of $SU(3)_L \times SU(3)_R$. So we see that the simple group theory can arise from one copy of QCD with 3 flavors in the vector limit! What was an ultraviolet W gauge boson in the original theory is a ρ meson in this QCD model, but by construction, both theories have the same low energy physics.

Finally, we can interpret eq. (45) as a picture of an AdS$_5$ space where the left-most site is associated with the UV brane and the right-most site is associated with the IR brane.

	Gauge Symmetry
UV Brane	$SU(2)$
Bulk	$SU(3)$
IR Brane	$SU(2)$

(51)

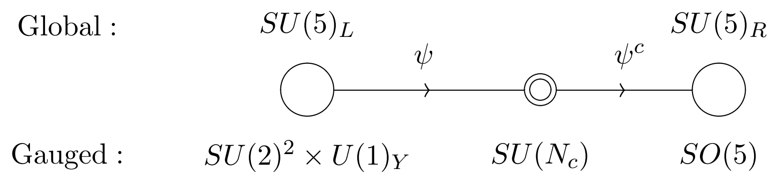
Modulo hypercharge, this symmetry pattern is exactly the symmetry pattern of the original realization of the Higgs as a holographic pseudo-Goldstone boson [23]! Like the $(SU(3)/SU(2))^2$ simple group, the model in [23] does not have a natural mechanism for generating a large Higgs quartic coupling, and this is to be expected because both theories have the same low energy degrees of freedom. Of course the $(SU(4)/SU(3))^2$ simple group [9] does have a way to generate a large Higgs quartic coupling, so the corresponding AdS_5 model with bulk $SU(4)$ gauge bosons broken to (misaligned) $SU(3)$ subgroups on the UV and IR branes could be a viable model of a composite Higgs boson.

But whereas we call the simple group theory a "little Higgs" theory, generic implementations of the Higgs as a holographic pseudo-Goldstone boson are not (currently) referred to as little Higgs theories. As we mentioned at the beginning of this section, though, *any* AdS_5 model with an electroweak doublet Goldstone can be a little Higgs theory because there is always a notion of collective breaking, namely collectively choosing boundary conditions on the UV and IR branes. In terms of the two-site moose from eq. (33), collective breaking means that both the F_μ and H_μ gauge couplings must be turned on in order for the uneaten Goldstones to acquire a radiative potential, and when we interpret the H_μ gauge fields as ρ-like mesons from a confining theory, there is a useful notion of collective breaking to the extent that the ρs are light (*i.e.* the H_μ gauge coupling is small, or equivalently, we are in the vector limit of the confining theory).

In other words, what makes an interesting theory of a composite Higgs boson is not collective breaking *per se*, but whether the Higgs potential can successfully trigger electroweak symmetry breaking while maintaining a large hierarchy between the confinement scale and the electroweak scale. Any old symmetry breaking pattern that yields an electroweak doublet Goldstone will be a little Higgs theory if there are light ρ mesons in the spectrum, but only very special theories can generate a light Higgs with a large quartic coupling, the necessary ingredients for a successful composite Higgs model.

6 Fermions and the Moose-ified Littlest Higgs

The $SU(5)/SO(5)$ littlest Higgs [5] is such a model with a naturally large quartic coupling. To construct a realistic littlest Higgs theory, we have to include the fermion sector of the standard model. In this section, we sketch how to incorporate the top quark into the $N_f = 5$ QCD UV completion of the littlest Higgs from eq. (5):



(52)

When $SU(N_c)$ confines, $\langle\psi\psi^c\rangle$ condenses to become the link field ξ. We will see that while it is certainly possible to include standard model fermions in the moose-ified littlest Higgs, we will run into many of the same challenges from technicolor in trying to generate a large enough top Yukawa coupling.

In the original language of global $SU(5)/SO(5)$ breaking [5], the relevant Goldstones were packaged into the field $\Sigma = \xi^2\Sigma_0$, where

$$\Sigma_0 \equiv \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}. \tag{53}$$

Σ transforms as $\Sigma \to V\Sigma V^T$, where $V \in SU(5)$. The Higgs doublet comprises four of the Goldstones in ξ. We will use the gauge sector and fermion content of [14]. In that case, there is an $SU(2)^2 \times U(1)_A$ subgroup of $SU(5)$ generated by

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & & \\ & & \\ & & \end{pmatrix}, \qquad Q_2^a = \begin{pmatrix} & & \\ & & \\ & & -\sigma^{*a}/2 \end{pmatrix}, \qquad A = \mathrm{diag}(1,1,0,-1,-1)/2. \tag{54}$$

Both $SU(2)$ subgroups are gauged, and the breaking of $SU(5) \to SO(5)$ Higgses $SU(2)^2$ to the diagonal $SU(2)_{EW}$. There is a separate $U(1)_B$ group, and hypercharge is generated by $Y = A + B$.

The fermion lagrangian of the littlest Higgs can be written compactly as

$$\mathcal{L} = f\left(\lambda_1 Q\Sigma^\dagger Q^c + \lambda_2 \tilde{Q}\Sigma_0 Q^c + \lambda_3 Q\Sigma_0 \tilde{Q}^c + h.c.\right), \tag{55}$$

where f is the decay constant of ξ, and the fermions transform as:

	$SU(3)_C$	$SU(5)$	$U(1)_B$
Q	$\mathbf{3}$	$\mathbf{5}$	$+2/3$
Q^c	$\bar{\mathbf{3}}$	$\mathbf{5}$	$-2/3$
$\tilde{Q}$	$\mathbf{3}$	$\bar{\mathbf{5}}$	$+2/3$
$\tilde{Q}^c$	$\bar{\mathbf{3}}$	$\bar{\mathbf{5}}$	$-2/3$

(56)

where $SU(3)_C$ is the color gauge group, and tildes have nothing to do with the $\mathbf{Z}_2$ automorphism from Section 3. As written, the lagrangian in eq. (55) does not break any of the $SU(5)$ symmetries that protect the Higgs. However, $SU(5)$ is explicitly broken because $\tilde{Q}$ and $\tilde{Q}^c$ are incomplete $SU(5)$ multiplets:

$$Q = \begin{pmatrix} p \\ \tilde{t} \\ q \end{pmatrix}, \qquad Q^c = \begin{pmatrix} \tilde{q}^c \\ t^c \\ p^c \end{pmatrix}, \qquad \tilde{Q} = \begin{pmatrix} 0 \\ 0 \\ \tilde{q} \end{pmatrix}, \qquad \tilde{Q}^c = \begin{pmatrix} 0 \\ \tilde{t}^c \\ 0 \end{pmatrix}. \tag{57}$$

In order for the Higgs to acquire a radiative potential from the top sector, each λ_i must be non-zero, and therefore eq. (55) manifestly exhibits collective breaking. (See [14, 24] for a more detailed discussion of the structure and purpose of this fermion content.) Couplings to other standard model fermions can be included by adding explicit $SU(5)$ violating couplings to Σ.

To simplify the discussion a bit, we will work in a slightly different $SU(5)$ basis where $\Sigma_0 = \mathbb{1}$. The unbroken $SO(5)$ generators T_a and the broken $SU(5)/SO(5)$ generators X_a satisfy

$$T_a = -T_a^T, \qquad X_a = X_a^T. \tag{58}$$

We see readily that $SU(5)/SO(5)$ is a symmetric space where the $\mathbf{Z}_2$ automorphism is $Q_a \to -Q_a^T$. Note that $\xi = e^{i\pi_a X_a}$, so $\xi = \xi^T$. For this reason, is convenient to make explicit the implied transposes in eq. (55):

$$\mathcal{L} = f\left(\lambda_1 Q^T \xi^* \xi^\dagger Q^c + \lambda_2 \tilde{Q}^T Q^c + \lambda_3 Q^T \tilde{Q}^c + h.c.\right). \tag{59}$$

We are already anticipating that ξ will transforms as $\xi \to L\xi R^\dagger$ under $SU(5)_L \times SU(5)_R$ as suggested by eq. (52). Note that $\xi\xi^T$ is indeed invariant under $SO(5)_R$, which is necessary for the gauge invariance of the QCD moose.

There are three simple ways to embed the fermions from eq. (59) into the QCD moose. The first is to simply identify the original $SU(5)$ with $SU(5)_L$, in which case the λ_1 interaction would arise from the dimension nine operator

$$\frac{1}{M_{\mathrm{NP}}^5} Q^T (\psi\psi^c)^* (\psi\psi^c)^\dagger Q^c, \tag{60}$$

where we are imagining that there is some new physics at M_{NP} that generates this interaction. This is a highly irrelevant operator, and unless we postulate some large anomalous dimension for the condensate $\langle\psi\psi^c\rangle$ [44], it seems highly unlikely to ever end up with an $\mathcal{O}(1)$ λ_1 if M_{NP} is much different from Λ_{QCD}.

Another way to include fermions is to take a cue from the AdS$_5$ construction of [24]. In that case, Q and Q^c are bulk fermions, so in the moose of eq. (52), we should have Q and Q^c fermions charged under $SU(5)_L$ and χ and χ^c fermions charged under $SU(5)_R$ with the same $SU(3)_C$ and $U(1)_B$ charges as their counterparts. The generalization of eq. (59) is

$$\mathcal{L} = f\left(\lambda_{1a} Q^T \xi^* \chi^c + \lambda_{1b} \chi^T \xi^\dagger Q^c + \lambda_{1c} \chi^T \chi^c + \lambda_2 \tilde{Q}^T Q^c + \lambda_3 Q^T \tilde{Q}^c + h.c.\right), \tag{61}$$

which is invariant under $SU(5)_L \times SU(5)_R$ when $SU(5)_R$ is restricted to $SO(5)_R$. Integrating out χ and χ^c, we see that $\lambda_1 = \lambda_{1a}\lambda_{1b}/\lambda_{1c}$. In terms of the fermions ψ and ψ^c, we could generate the appropriate four fermion operators through an extended technicolor-like mechanism [33, 34]:

$$\frac{(4\pi)^2}{M_{\mathrm{ETC}}^2} Q^T (\psi\psi^c)^* \chi^c, \qquad \frac{(4\pi)^2}{M_{\mathrm{ETC}}^2} \chi^T (\psi\psi^c)^\dagger Q^c, \qquad m_\chi \chi^T \chi^c, \tag{62}$$

where we are assuming that all gauge couplings are large. If the condensate $\langle\psi\psi^c\rangle$ takes its NDA value $4\pi f^3$, then parametrically

$$\lambda_1 \sim \frac{4\pi \Lambda_{\mathrm{QCD}}^5}{M_{\mathrm{ETC}}^4 m_\chi}, \qquad \Lambda_{\mathrm{QCD}} \sim 4\pi f. \tag{63}$$

In order to have $\lambda_1 \sim \mathcal{O}(1)$, the ETC scale must be quite low or the $\langle\psi\psi^c\rangle$ must have a large anomalous dimension. Also, to have a realistic top Yukawa coupling and a light t' fermion partner, all of the λ_i values must be roughly the same, and in this scenario, there is no explanation for why the $\tilde{Q}Q^c$ and $Q\tilde{Q}^c$ mass terms will be roughly the same size as the $Q\Sigma^\dagger Q^c$ interaction.

Finally, we can rewrite eq. (59) equivalently

$$\mathcal{L} = f\left(\lambda_1 (\xi^\dagger Q)^T (\xi^\dagger Q^c) + \lambda_2 \tilde{Q}^T \xi (\xi^\dagger Q^c) + \lambda_3 (\xi^\dagger Q)^T \xi^T \tilde{Q}^c + h.c.\right). \tag{64}$$

This suggests that we could identify $\xi^\dagger Q \equiv Q'$ and $\xi^\dagger Q^c \equiv Q'^c$ as fundamentals of $SU(5)_R$:

$$\mathcal{L} = f\left(\lambda_1 Q'^T Q'^c + \lambda_2 \tilde{Q}^T \xi Q'^c + \lambda_3 Q'^T \xi^T \tilde{Q}^c + h.c.\right). \tag{65}$$

Though it might appear that Q' and Q'^c have lost information about their $SU(2)^2$ charges, recall that when we integrate out the $SO(5)$ gauge bosons, couplings to the gauged subgroup of $SU(5)_L$ are induced as in eq. (39). We can now use an ETC-like mechanism to generate the operators

$$\frac{(4\pi)^2}{M_{\text{ETC}}^2}\tilde{Q}^T(\psi\psi^c)Q'^c, \qquad \frac{(4\pi)^2}{M_{\text{ETC}}^2}Q'^T(\psi\psi^c)^T\tilde{Q}^c. \tag{66}$$

Again, without a low ETC scale or a large $\langle\psi\psi^c\rangle$ condensate vev, it will be difficult to generate a large enough top Yukawa coupling, and there is no reason why the λ_i values should be similar.

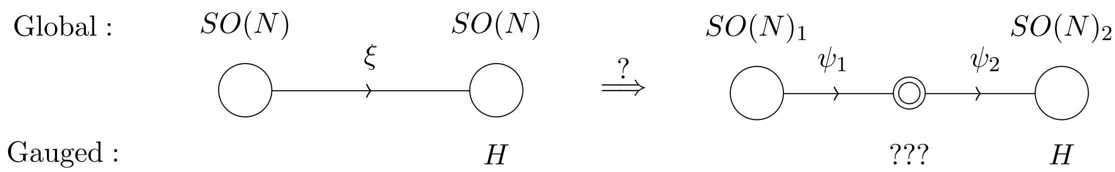
Of course, apart from the λ_i degeneracy problem, these difficulties in generating realistic Yukawa couplings are identical to issues in technicolor. To the extent that little Higgs theories predict perturbative physics well above the the electroweak scale, the moose-ified littlest Higgs is an improvement over technicolor in that it has a realistic chance to address precision electroweak constraints. The lack of a GIM mechanism [13] in extended technicolor means that the implementation of fermions presented in this section will most likely generate large sources of flavor violation unless we are able to raise the ETC scale via some walking mechanism. But in terms of the particle content of the littlest Higgs (if not the actual numeric spectrum of the theory), we have seen that it is fully consistent to embed standard model fermions in the $N_f = 5$ QCD UV completion of the littlest Higgs.

7 Future Directions

In this paper, we have explored a beautiful connection between an acronym soup of ideas that were already tantalizing related, but which become even more intertwined in the context of AdS/CFT. HLS is a model for the ρ mesons in QCD which is known to generate G/H flavor symmetry breaking that is phenomenologically described by CCWZ which is the key to constructing LH theories. AdS/CFT allows us to explicitly realize the vector limit (*i.e.* large N_c or light ρ meson limit) of a confining theory, and once we deconstruct the corresponding AdS$_5$ model, we are free to interpret the link fields either as Wilson lines constructed out of A_5, or as fermion condensates from a completely different QCD theory. In other words, we can interpret the light spin-1 degrees of freedom either as holographic ρ mesons or as ultraviolet W' gauge fields that acquire a mass via spontaneous symmetry breaking.

Note that there is no deep duality between the holographic CFT and the new QCD theory. All we have shown is that two different theories can have the same low energy degrees of freedom, and this is particularly interesting in the context of the little Higgs, where the relevant phenomenology depends almost entirely on the low energy spectrum. What is a bit surprising (but obvious in retrospect) is that one possible UV completion of an $(SU(N)/H)^n$ little Higgs is n copies of technicolor! We have known since [15, 16] that ordinary QCD could give masses to W and Z bosons through spontaneous symmetry breaking; now we know that ordinary QCD can give rise to an electroweak doublet pion which can subsequently break electroweak symmetry. This allows us (or dooms us, depending on your perspective) to use the full machinery of extended technicolor [33, 34] and walking technicolor [44] to address phenomenological questions in little Higgs theories.

What is not obvious is whether general $(G/H)^n$ symmetry breaking patterns can emerge out of moose-like confining theories. A two site moose with global $SU(N)$ sites looks like the pion non-linear sigma model of QCD, but in order to construct moose diagrams for $SO(N)/H$ theories (such as the $SO(9)/(SO(5) \times SO(4))$ model of [45]), we would have to find a UV completion for the moose link field:



(67)

In order to have $SO(N)$ flavor symmetries, the confining group must have real representations. To keep the $SO(N)^2$ flavor symmetry from enlarging to an $SO(2N)$ flavor symmetry, the confining group must have two different real representations $\mathbf{R_1}$ and $\mathbf{R_2}$. Furthermore, for $SO(N)^2$ to spontaneously break to $SO(N)_V$ via confinement, the most attractive condensate must be $\langle \mathbf{R_1 R_2} \rangle$. Similarly, to get an $Sp(N)^2 \to Sp(N)_V$ symmetry breaking pattern, we would need a confining group with two pseudoreal representations $\mathbf{P_1}$ and $\mathbf{P_2}$ which condense as $\langle \mathbf{P_1 P_2} \rangle$. Whether these situations can in fact be realized is an open question, though it seems highly unlikely for G to be one of the exceptional groups.

Interestingly, the only constraint on the subgroup H is anomaly cancelation, because we can weakly gauge any global symmetry we wish. This suggests a solution to vacuum alignment problems in certain little Higgs theories [ref?], because once we are convinced that it is possible to realize $G_1 \times G_2 \to G_V$ symmetry breaking, the low energy effective lagrangian for the G/H moose will be a G/H non-linear sigma model by construction, regardless of which H we choose. WRONG!

We have seen that the G/H moose in some sense trivializes the CCWZ prescription, and is interesting to wonder whether the same construction might trivialize another consequence of spontaneous symmetry breaking: the WZW term [36, 37]. The WZW term accounts for the fact that anomalies above the confinement scale must match anomalies in the low energy effective theory [46]. It is well-known that a Chern-Simons (CS) term in the bulk of AdS_5 can transmit anomalies from the UV brane to the IR brane [ref?], giving a nice 5D representation of anomaly matching in the holographic CFT. It is also well-known that the WZW term can be derived by considering a CS term in the interior of a 5-ball, with ordinary 4D space-time on the boundary of the ball [37, 47], though to concisely express the WZW only in terms of phenomenological variables requires the full machinery of differential geometry [48]. It is even known how to derive the WZW term using differential geometry in the language of HLS [49], so it should be possible to understand the WZW term in term of the G/H moose.

The difficulty is how to deconstruct the CS term in AdS_5, because by design, gauge transformations of the CS term introduce new terms on the UV and IR branes. Deconstructing a dimension usually involves going to $A_5 = 0$ gauge and then reintroducing Wilson line link fields, and in principle, one should track changes in the CS term through this whole procedure. In the context of

QCD, the deconstructed CS term has already been studied based on the following moose [50]:

$$\begin{array}{lccc} \text{Global :} & SU(N_f)_L & & SU(N_f)_R \\ & \bigcirc & \xrightarrow{\quad U \quad} & \bigcirc \\ \text{Gauged :} & SU(N_f)_L & & SU(N_f)_R \end{array} \tag{68}$$

where the gauged $SU(N_f)_i$ correspond to arbitary left- and right-handed currents. Now with inspiration from AdS/CFT, we want to understand the CS term in the G/H moose, where the natural variable is ξ, the "square root" of U:

$$\begin{array}{lccc} \text{Global :} & SU(N_f)_L \times SU(N_f)_R & & SU(N_f)_L \times SU(N_f)_R \\ & \bigcirc & \xrightarrow{\quad \xi \quad} & \bigcirc \\ \text{Gauged :} & SU(N_f)_L \times SU(N_f)_R & & SU(N_f)_V \end{array} \tag{69}$$

In pedagogical CCWZ, we saw that an essential step in constructing the Goldstone kinetic terms was integrating out the H_μ field (here, the $SU(N_f)_V$ gauge bosons), and we expect that this will also be an essential ingredient in constructing the moose-ified WZW term. If successful, we can construct a meta-acronym that ties together some of the most interesting ideas in gauge theories and G/H phenomenology: A(QCd)(CS)/(CC(WZW))FT HLS/LH.

Acknowledgments: Nima, Howard, Itay.

References

[1] D. J. Gross and F. Wilczek, *Ultraviolet Behavior Of Non-Abelian Gauge Theories,* Phys. Rev. Lett. **30**, 1343 (1973).

[2] H. D. Politzer, *Reliable Perturbative Results For Strong Interactions?,* Phys. Rev. Lett. **30**, 1346 (1973).

[3] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction,* Phys. Lett. B **513**, 232 (2001) [arXiv:hep-ph/0105239].

[4] N. Arkani-Hamed, A. G. Cohen, T. Gregoire and J. G. Wacker, *Phenomenology of electroweak symmetry breaking from theory space,* JHEP **0208**, 020 (2002) [arXiv:hep-ph/0202089].

[5] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, *The littlest Higgs,* JHEP **0207**, 034 (2002) [arXiv:hep-ph/0206021].

[6] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, *The minimal moose for a little Higgs,* JHEP **0208**, 021 (2002) [arXiv:hep-ph/0206020].

[7] T. Gregoire and J. G. Wacker, *Mooses, topology and Higgs,* JHEP **0208**, 019 (2002) [arXiv:hep-ph/0206023].

[8] I. Low, W. Skiba and D. Smith, *Little Higgses from an antisymmetric condensate,* Phys. Rev. D **66**, 072001 (2002) [arXiv:hep-ph/0207243].

[9] D. E. Kaplan and M. Schmaltz, *The little Higgs from a simple group,* JHEP **0310**, 039 (2003) [arXiv:hep-ph/0302049].

[10] W. Skiba and J. Terning, *A simple model of two little Higgses,* Phys. Rev. D **68**, 075001 (2003) [arXiv:hep-ph/0305302].

[11] H. C. Cheng and I. Low, *TeV symmetry and the little hierarchy problem,* JHEP **0309**, 051 (2003) [arXiv:hep-ph/0308199].

[12] H. C. Cheng and I. Low, *Little hierarchy, little Higgses, and a little symmetry,* arXiv:hep-ph/0405243.

[13] S. L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions With Lepton-Hadron Symmetry,* Phys. Rev. D **2**, 1285 (1970).

[14] E. Katz, J. y. Lee, A. E. Nelson and D. G. E. Walker, *A composite little Higgs model,* arXiv:hep-ph/0312287.

[15] S. Weinberg, *Implications Of Dynamical Symmetry Breaking,* Phys. Rev. D **13**, 974 (1976).

[16] L. Susskind, *Dynamics Of Spontaneous Symmetry Breaking In The Weinberg-Salam Theory,* Phys. Rev. D **20**, 2619 (1979).

[17] J. M. Maldacena, *The large N limit of superconformal field theories and supergravity,* Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[18] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Gauge theory correlators from non-critical string theory,* Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].

[19] E. Witten, *Anti-de Sitter space and holography,* Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[20] N. Arkani-Hamed, M. Porrati and L. Randall, *Holography and phenomenology,* JHEP **0108**, 017 (2001) [arXiv:hep-th/0012148].

[21] R. Rattazzi and A. Zaffaroni, *Comments on the holographic picture of the Randall-Sundrum model,* JHEP **0104**, 021 (2001) [arXiv:hep-th/0012248].

[22] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension,* Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[23] R. Contino, Y. Nomura and A. Pomarol, *Higgs as a holographic pseudo-Goldstone boson,* Nucl. Phys. B **671**, 148 (2003) [arXiv:hep-ph/0306259].

[24] J. Thaler and I. Yavin, *The Littlest Higgs in Anti-de Sitter Space,* arXiv:hep-ph/0501036.

[25] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *(De)constructing dimensions,* Phys. Rev. Lett. **86**, 4757 (2001) [arXiv:hep-th/0104005].

[26] L. Randall, Y. Shadmi and N. Weiner, *Deconstruction and gauge theories in AdS(5),* JHEP **0301**, 055 (2003) [arXiv:hep-th/0208120].

[27] S. R. Coleman, J. Wess and B. Zumino, *Structure Of Phenomenological lagrangians. 1,* Phys. Rev. **177**, 2239 (1969).

[28] C. G. Callan, S. R. Coleman, J. Wess and B. Zumino, *Structure Of Phenomenological lagrangians. 2,* Phys. Rev. **177**, 2247 (1969).

[29] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, *Is Rho Meson A Dynamical Gauge Boson Of Hidden Local Symmetry?,* Phys. Rev. Lett. **54**, 1215 (1985).

[30] M. Bando, T. Fujiwara and K. Yamawaki, *Generalized Hidden Local Symmetry And The A1 Meson,* Prog. Theor. Phys. **79**, 1140 (1988).

[31] M. Bando, T. Kugo and K. Yamawaki, *Nonlinear Realization And Hidden Local Symmetries,* Phys. Rept. **164**, 217 (1988).

[32] M. Piai, A. Pierce and J. Wacker, *Composite vector mesons from QCD to the little Higgs,* arXiv:hep-ph/0405242.

[33] E. Eichten and K. D. Lane, *Dynamical Breaking Of Weak Interaction Symmetries,* Phys. Lett. B **90**, 125 (1980).

[34] S. Dimopoulos and L. Susskind, *Mass Without Scalars,* Nucl. Phys. B **155**, 237 (1979).

[35] H. Georgi, *Vector Realization Of Chiral Symmetry,* Nucl. Phys. B **331**, 311 (1990).

[36] J. Wess and B. Zumino, *Consequences Of Anomalous Ward Identities,* Phys. Lett. B **37**, 95 (1971).

[37] E. Witten, *Global Aspects Of Current Algebra,* Nucl. Phys. B **223**, 422 (1983).

[38] G. 't Hooft, *A Planar Diagram Theory For Strong Interactions,* Nucl. Phys. B **72**, 461 (1974).

[39] A. Manohar and H. Georgi, *Chiral Quarks And The Nonrelativistic Quark Model,* Nucl. Phys. B **234**, 189 (1984).

[40] H. Georgi and L. Randall, *Flavor Conserving CP Violation In Invisible Axion Models,* Nucl. Phys. B **276**, 241 (1986).

[41] C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, *Gauge theories on an interval: Unitarity without a Higgs,* Phys. Rev. D **69**, 055006 (2004) [arXiv:hep-ph/0305237].

[42] K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model,* arXiv:hep-ph/0412089.

[43] S. Chang and H. J. He, *Unitarity of little Higgs models signals new physics of UV completion,* Phys. Lett. B **586**, 95 (2004) [arXiv:hep-ph/0311177].

[44] B. Holdom, *Raising The Sideways Scale,* Phys. Rev. D **24**, 1441 (1981).

[45] S. Chang, *A 'littlest Higgs' model with custodial SU(2) symmetry,* JHEP **0312**, 057 (2003) [arXiv:hep-ph/0306034].

[46] G. 't Hooft, in *Recent Developments In Gauge Theories: Nato Advanced Study Institute, Cargese (1979),* (Plenum, New York, 1980).

[47] B. Zumino, Y. S. Wu and A. Zee, *Chiral Anomalies, Higher Dimensions, And Differential Geometry,* Nucl. Phys. B **239**, 477 (1984).

[48] J. L. Mañes, *Differential Geometric Construction Of The Gauged Wess-Zumino Action,* Nucl. Phys. B **250**, 369 (1985).

[49] Y. S. Wu, *Local Cohomology Of Gauge Fields And Explicit Construction Of Wess-Zumino Lagrangians In Nonlinear Sigma Models Over G/H,* Phys. Lett. B **153**, 70 (1985).

[50] C. T. Hill and C. K. Zachos, *Dimensional deconstruction and Wess-Zumino-Witten terms,* arXiv:hep-th/0411157.