PREPRINT

Spontaneous Lorentz Breaking at High Energies

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Abstract

Theories that spontaneously break Lorentz invariance inevitably also violate diffeomorphism symmetries, which implies the existence of extra degrees of freedom and modifications of gravity. As a result, the scale at which Lorentz invariance could be broken is constrained by experimental observations in the gravity sector. In the simplest example of ghost condensation which contains only one extra degree of freedom at low energies, the scale of Lorentz violation M can not be much higher than $M \sim 10$ MeV due to an infrared instability in the gravity sector. We show that this bound may be relaxed in theories containing more degrees of freedom, in particular, if Lorentz invariance is broken via the vacuum expectation value of a vector field. Such theories can be viewed as gauged versions of ghost condensation, and thus have a systematic effective field theory description that allows us to estimate the size of all physical effects. The strongest bounds on the gravitational sector come from direction-dependent gravitational forces, and imply that $M \lesssim 10^{15}$ GeV. In coupling to matter, the transverse modes of the vector field can induce spin-dependent inverse-square law forces in the same way as the Goldstone mode, but with a different angular characteristic, and there are additional novel effects that are absent in ghost condensation with only the Goldstone mode. These provide new possible experimental signatures for Lorentz violation and allow these theories to be distinguished from (ungauged) ghost condensation.

1 Introduction

As we approach the 100th anniversary of Special Relativity, Lorentz invariance reigns as (arguably) the most important symmetry in modern physics. Perhaps the best way to understand the central role of Lorentz invariance is to imagine ways that it might be violated. Like baryon and lepton numbers in the Standard Model, Lorentz invariance could be an accidental symmetry that just happens to be valid for the leading interactions in quantum field theory. Alternatively, Lorentz symmetry could simply be one limit of a more fundamental symmetry, in the same way that the Galilean group is the small velocity limit of the Lorentz group. But in the absence of experimental data, it is difficult to guess what deeper organizing principle or more fundamental symmetry could replace Lorentz invariance. Indeed, the Lorentz group itself is but one of many possible deformations of the Galilean group, and it was the experimental crisis of the ether that really drove the transition from absolute time to the space-time continuum in 1905.

In this paper, we focus on a less radical possibility: that Lorentz invariance is a good symmetry of the universe at high energies but is spontaneously broken by the vacuum at an energy scale M. There are two experimental consequences of such a scenerio. First, if the sector that spontaneously breaks Lorentz symmetry is coupled to the Standard Model, then we will see a range of explicit Lorentz-violating operators, and there is a large literature studying such Lorentz-violating extensions of the Standard Model (see Ref. [1, 2, 3] for examples and further references). Second, as shown in Ref. [4] and emphasized in Ref. [5], if the Lorentz group is spontaneously broken to $SO(3)$ rotations, then in addition to Lorentz-violating operators involving just Standard Model fields, there will be a new massless degree of freedom π that gives rise to dynamical Lorentz-violating effects. The field π is the Goldstone boson of spontaneous Lorentz violation, and in a gravitational theory, π is also the Goldstone boson of broken time diffeomorphisms and thus mixes with the metric. Therefore, spontaneous Lorentz violation is intimately tied to modifications of gravity, so even in the absence of direct couplings between a Lorentz-violating sector (such as the ghost condensate) and the Standard Model, we can derive bounds on Lorentz violations simply by looking at gravitational physics.

In fact, there are strong constraints on the symmetry breaking scale M coming from mixing between π and the Newtonian potential. This mixing produces an $\mathcal{O}(1)$ modification to Newton's Law at length scales greater than $r_c = M_{\rm Pl}/M^2$ and time scales longer than $t_c = M_{\rm Pl}^2/M^3$. If we demand no modification to Newton's Law over the lifetime of the universe, then we are forced to take $M \lesssim 10$ MeV. This is exactly analogous to spontaneously symmetry breaking in non-Abelian gauge theories; the mass of the gauge boson m and the gauge coupling g are related to the Goldstone decay constant f by $f = m/g$, and if we insist that the gauge boson mass is smaller than a certain scale, then for fixed g we also bound the Goldstone decay constant f and hence the scale $\Lambda \sim 4\pi f$ where new physics must enter to unitarize Goldstone-Goldstone scattering. In the case of spontaneous Lorentz violation, $1/t_c$ plays roughly the role of the mass for gravity, and M is the scale where π-π scattering violates unitarity. In other words, in order for the graviton "mass" to be smaller than the Hubble scale, there must be new physics entering at around the electron mass scale.

If we take this logic seriously, it at first seems impossible to raise the symmetry breaking scale M much higher than 10 MeV. The Goldstone boson π is a necessary consequence of spontaneous Lorentz violation, and in order to have a diffeomorphically invariant low-energy effective theory, the kinetic terms for π necessarily involve mixing with the Newtonian potential. Said another way, we can go to a unitary gauge where we use the diffeomorphism redundancy to set $\pi \equiv 0$, and in that gauge there is a mass term for the Newtonian potential that cannot be removed. Now, this mass term is far less dangerous than in other theories that modify gravity at long distances. We can characterize the strong coupling scale of a modified gravitational theory by $\Lambda_n = (m_{\rm grav}^{n-1} M_{\rm Pl})^{1/n}$, where $m_{\rm grav}$ is either the graviton mass or simply the inverse length or time scale when Newton's law is significantly modified. As n increases, the separation between the graviton mass and the strong coupling scale decreases, indicating that the theory needs an ultraviolet completion at a lower scale. In Fierz-Pauli massive gravity, the strong coupling scale is Λ_5, though in a non-linear extension of Fierz-Pauli, the scale can be pushed to Λ_3 [6]. DGP massive gravity [7] also has a strong coupling scale of Λ_3 [8], and a recent proposal by Rubakov involving Lorentz-violating mass terms has a strong coupling scale Λ_2, assuming one fine-tuning condition [9] (see also Ref. [10], where the fine-tuning can be guaranteed by certain symmetries). If we take $m_{\rm grav} = 1/t_c$, then the theory of spontaneous Lorentz violation has a strong coupling scale $M = \Lambda_{1.5}$, better than other theories of modified gravity but nowhere near $\Lambda_1 = M_{\rm Pl}$.

So we pose the question: can we spontaneously break Lorentz invariance at high scales? Amazingly, we find the answer is yes. The key is to introduce a new $U(1)$ gauge symmetry, and if the gauge coupling g is larger than the critical gauge coupling $g_c \sim M/M_{\rm Pl}$, then we can raise the scale M as high as 10^{15} GeV without coming into conflict with precision tests of gravity. We will see that introducing this $U(1)$ corresponds to "gauging" ghost condensation. More intuitively, the role of the $U(1)$ gauge symmetry is to "eat" the Goldstone field π. That is, the symmetry breaking pattern is time diffeomorphisms cross the new $U(1)$ breaking down to the vector subgroup, and π is the Goldstone boson that nonlinearly realizes the axial subgroup. As we raise the gauge coupling g, the axial subgroup becomes mostly the $U(1)$ gauge symmetry, so π dominantly mixes with the $U(1)$ field and not with the Newtonian potential.

In other words, because the graviton universally couples to matter but the $U(1)$ gauge boson can safely live in a hidden sector, we can dial the gauge coupling to hide Lorentz violations from gravity and the Standard Model, assuming no Standard Model fields are charged under the new $U(1)$ gauge group. We will see explicitly that for $g > g_c$, an effective mass term for the graviton is forbidden, and the modification of gravity in this model is dominantly just a redefinition of Newton's constant. Because we break Lorentz invariance, there are also preferred frame effects in gravity, and it turns out that these effects will put the strongest bounds on the scale M. So while we cannot push the scale of spontaneous Lorentz violation all the way to the Planck scale, we can raise the scale M well above 10 MeV and into scales relevant for particle physics.

Also, we note that the theory of gauged ghost condensation is really just a theory where a vector A_μ gets a vacuum expectation value in a time-like direction. In the $g \to 0$ limit, the vector A_μ can be replaced by its longitudinal component $\partial_\mu \phi$, reducing the theory to original ghost condensation. There have been many studies of gravitational theories with non-zero $\langle A_\mu \rangle$, beginning with vector-tensor theories of gravity in the 1970s [11, 12]. There are also more recent studies in Ref. [13, 14]. Our goal is to understand these theories in the framework of effective field theory, illuminating the power counting to understand why the modification of gravity is generically small for large enough gauge coupling. We explore the specifics of how the gravity sector is modified, taking into account non-linear effects, and also study some of the experimental consequences of possible couplings between A_μ and the Standard Model. We will see that the signatures for gauged ghost condensation are very different from the ungaged case, suggesting new ways to search for spontaneous Lorentz violation within a class of theories where modifications to gravity are supressed.

2 A Bottom-Up View of High Scale Lorentz Violation

Before we show the construction of gauged ghost condensation, it is instructive to see from the bottom-up how we are able to forbid a mass term for the Newtonian potential even in the presence of explicit Lorentz-violating interactions. We will review the argument from Ref. [5] that breaking Lorentz-invariance implies such mass terms, and then show how a simple modification of this argument leads to a theory where gravity masses are forbidden. At the end of this section, we express this modification in a more convenient form that will make the connection to "gauging" ghost condensation obvious.

Let us consider the simplest way to break Lorentz invariance by inserting a time-component of a vector operator into the Lagrangian,

$$\Delta\mathcal{L} = J^0. \tag{2.1}$$

This term breaks Lorentz symmetry but preserves $SO(3)$ spatial rotations. More importantly, this term also breaks diffeomorphism invariance. To see this, we expand the metric around flat space,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{2.2}$$

The diffeomorphism generated by the coordinate transformation $x^\mu \to x^\mu + \xi^\mu$ acts on the linearized metric $h_{\mu\nu}$ and a vector J_μ to leading order as

$$\begin{aligned} h_{\mu\nu} &\to h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \\ J^\mu &\to J^\mu - (\partial^\mu \xi_\nu) J^\nu - \xi^\nu \partial_\nu J^\mu. \end{aligned} \tag{2.3}$$

It is easy to check that there is no way to insert factors of $h_{\mu\nu}$ into $\Delta\mathcal{L} = J^0$ to make this term diffeomorphically invariant. The fact that diffeomorphisms are broken is not a conceptual problem. Just like in spontaneously broken gauge symmetries, the absence of gauge invariance at low energies simply tells us that the gauge fields are propagating additional degrees of freedom, namely the Goldstone bosons for the broken symmetry. The only confusing point in this case is that the Goldstone boson(s) of spontaneous Lorentz violation are the same as the Goldstone boson(s) of spontaneous diffeomorphism breaking.

Indeed, we can isolate the new Goldstone degree of freedom by introducing a Goldstone field π that transforms as a shift under time diffeomorphisms,

$$\pi \to \pi - \xi_0. \tag{2.4}$$

We can then make the interaction in Eq. (2.1) diffeomorphically invariant with the help of the π field,

$$\Delta\mathcal{L} = \sqrt{g}(J^0 - \partial_\mu \pi J^\mu). \tag{2.5}$$

In order to make π a healthy propagating degree of freedom, we have to introduce kinetic terms for it. However, we see that a diffeomorphically invariant time-kinetic piece for π involves mixing with the metric

$$\mathcal{L}_{\text{time-kinetic}} = \frac{M^4}{2}\left(\dot{\pi} - \frac{1}{2}h_{00}\right)^2, \tag{2.6}$$

where M is the scale associated with spontaneous Lorentz violation. Going to the unitary gauge ($\pi \equiv 0$), this becomes a mass term for the Newtonian potential $\Phi = h_{00}/2$. In fact, if the kinetic term for π has the right sign, the mass-squared for Φ has the wrong sign, implying an oscillatory (as opposed to Yukawa) modification to Newton's law. This modification is studied more carefully in Ref. [4], and becomes large at distances greater than $r_c = M_{\rm Pl}/M^2$ and for times longer than $t_c = M_{\rm Pl}^2/M^3$. Therefore, it appears that in order to have a diffeomorphically invariant low energy effective theory that describe spontaneous Lorentz violation, we are forced to introduce a mass term for $h_{\mu\nu}$.

However, what this argument fails to capture is that the π field is not the only way to restore diffeomorphisms to a Lorentz-violating Lagrangian. For example, we could have tried introducing three fields ψ_i that shift under spatial diffeomorphisms

$$\psi_i \to \psi_i - \xi_i. \tag{2.7}$$

While it is indeed possible to use the ψ_i fields to make Eq. (2.1) diffeomorphically invariant, it turns out that there is no way to generate healthy time-kinetic pieces for ψ_i without breaking time diffeomorphism invariance. Also, we could introduce Goldstone bosons for all four diffeomorphism directions and use a fine-tuning constraint (à la Ref. [9]) to make sure that one of combination of the Goldstones does not propagate. However, even with such a fine-tuning condition the graviton will get a mass of order $M^2/M_{\rm Pl}$. What we want is a way to introduce Goldstones in a way that the diffeomorphism invariance *forbids* a mass term for gravity of any kind.

The solution is to introduce three fields χ_i that shift as the *gradient* of time diffeomorphisms,

$$\chi_i \to \chi_i - \partial_i \xi_0. \tag{2.8}$$

(We will see that only the longitudinal mode of χ_i is a true Goldstone boson.) We can then complete Eq. (2.1) to the diffeomorphically invariant combination

$$\Delta\mathcal{L} = J^0 - h_{0i}J^i + \frac{1}{2}h^i_i J^0 + \chi_i J^i, \tag{2.9}$$

where a sum over the spatial indices i is implied using $\eta^{ij} = -\delta^{ij}$. The time kinetic piece for χ_i comes from

$$\mathcal{L}_{\rm time-kinetic} = -\frac{M^2}{2g^2}\left(\dot{\chi}_i - \frac{1}{2}\partial_i h_{00}\right)^2, \tag{2.10}$$

where the overall minus sign is chosen to give χ_i the right sign kinetic terms, and for now g is just some unknown coefficient. As one might guess, g will turn into the $U(1)$ gauge coupling mentioned in the introduction.

We see readily that no mass term for the Newtonian potential is generated by Eq. (2.10)! It is easy to check that all diffeomorphically invariant combinations of χ_i and $h_{\mu\nu}$ involve derivatives acting on the metric, so the modification of Newton's law is suppressed. In the non-relativistic limit, the χ_i time kinetic piece simply gives rise to a redefinition of Newton's constant. In terms of the Newtonian potential $\Phi = h_{00}/2$, the non-relativistic Einstein-Hilbert action is

$$\mathcal{L}_{\rm EH} = -M_{\rm Pl}^2(\vec{\nabla}\Phi)^2. \tag{2.11}$$

In the same limit, Eq. (2.10) becomes

$$\mathcal{L}_{\rm time-kinetic} = \frac{M^2}{2g^2}(\vec{\nabla}\Phi)^2, \tag{2.12}$$

so the effective Newton's constant G_N can be written in terms of the original Newton's constant G_N^0 as

$$G_N = \frac{G_N^0}{1-\epsilon^2}, \qquad \epsilon = \frac{M}{gM_{\rm Pl}\sqrt{2}}. \tag{2.13}$$

So as long as the unknown coefficient g is greater than $g_c = M/\sqrt{2}M_{\rm Pl}$, the effective Newton's constant is positive, and any modification of gravity will only come through kinetic mixing with χ_i.

Note that unlike the case of the π field, χ_i cannot be entirely removed by a diffeomorphism gauge choice. Only the longitudinal mode of χ_i can be removed by the choice of gauge, strongly hinting that the two transverse modes of χ_i should be thought of not as Goldstone bosons but as the two polarizations of a new $U(1)$ gauge field. Indeed, we can arrive at Eq. (2.8) by considering a Goldstone π that transforms as a shift under both time diffeomorphisms *and* a new $U(1)$ gauge symmetry:

$$\begin{aligned} \pi &\to \pi - \xi_0 - \frac{\lambda}{M}, \\ A_\mu &\to A_\mu + \partial_\mu \lambda, \end{aligned} \tag{2.14}$$

where the scale M is inserted to give π dimensions of length. We can think of the field χ_i as the gauge invariant combination

$$\chi_i = \frac{A_i}{M} + \partial_i \pi, \tag{2.15}$$

which clearly shifts the same way as Eq. (2.8).

What about the combination $\chi_0 = A_0/M + \partial_0 \pi$? Somehow this degree of freedom must be removed from the effective χ_i Lagrangian. Indeed, it is natural for χ_0 to be massive, because there is no symmetry forbidding the mass term

$$\mathcal{L}_{\rm mass} = \frac{M^4}{2}\left(\chi_0 - \frac{1}{2}h_{00}\right)^2. \tag{2.16}$$

We can use the diffeomorphism/gauge redundancy to go to a gauge where $\pi \equiv 0$. Going to canonical normalization for A_0 and the Newtonian potential Φ, Eq. (2.16) takes the form

$$\mathcal{L}_{\rm mass} = \frac{M^2}{2}\left(g A_0^c - g_c \Phi^c\right)^2, \quad A_0^c = \frac{A_0}{g}, \quad \Phi^c = \sqrt{2}M_{\rm Pl}\Phi, \quad g_c = \frac{M}{\sqrt{2}M_{\rm Pl}}. \tag{2.17}$$

We see that the combination $g A_0^c - g_c \Phi^c$ gets a mass of order $M\sqrt{g^2 + g_c^2}$, and if $g > g_c$, the massive mode is mostly A_0. Therefore, at energies below $\sim gM$, we can simply integrate out A_0!

The effect of integrating out A_0 is to introduce kinetic mixing between A_i and the Newtonian potential. The field A_0 appears in the gauge kinetic terms as

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} \supset \frac{1}{2}\left(\partial_0 A_i^c - \partial_i A_0^c\right)^2, \tag{2.18}$$

where we have gone to canonical normalization for A_μ. Setting A_0 to its low energy equation of motion $A_0^c = \epsilon\Phi^c$ with $\epsilon = g_c/g$,

$$\mathcal{L}_{\text{gauge}} \supset \frac{1}{2}\left(\partial_0 A_i^c - \epsilon\partial_i\Phi^c\right)^2 . \tag{2.19}$$

This is identical to Eq. (2.10) in the appropriate normalization. The parameter ϵ sets the mixing between A_i and the Newtonian potential, and because g is a free parameter in the theory, we can dial ϵ to be pretty much as small as we want as long as $g_c \sim M/M_{\text{Pl}}$ is not too large. As we will see when we analyze this theory more carefully, the gravitational physics will depend on g and g_c independently and not just through the combination $\epsilon = g_c/g$. Therefore, we can find a bound on M/M_{Pl} by looking at precision tests of gravity, and this will turn out to be the strongest bound on how high we can push the scale of spontaneous Lorentz violation.

One way of understanding why a mass term is forbidden in this model is to realize that in the Lorentz invariant theory of linearized gravity, the four diffeomorphisms ξ^μ protect a graviton mass term, and if any of these diffeomorphisms are broken, there is at least one quadratic term we can write down involving $h_{\mu\nu}$ with no derivatives,[1] for example, h_{00}^2 in the case of broken time diffeomorphisms. However, there is no reason why a linear combination of time diffeomorphisms and a $U(1)$ gauge symmetry cannot be preserved, so by introducing the $U(1)$ we have in effect introduced a fourth "diffeomorphism" to protect the graviton mass. This explains why in the models considered in Ref. [13], the modification of gravity is so moderate. Of course, we now understand that it is not enough to simply introduce a $U(1)$ gauge symmetry; we also have to raise the gauge coupling high enough so that the preserved "diffeomorphism" symmetry is mostly ξ^0.

Other possibilities: Rubakov, Dubovsky...

3 Gauged Ghost Condensation

The discussion of the previous section has suggested that the $U(1)$ extension of spontaneous Lorentz violation can be described by a gauged version of ghost condensation. In ghost condensation, the derivative of a scalar field $\partial_\mu\phi$ took a vacuum expectation value in a time-like direction; in gauged ghost condensation we simply replace the ordinary derivative by the gauge covariant derivative. In this section, we describe how to construct the effective theory of the gauged ghost condensation by first turning off gravity, with an emphasis on the power counting and the size of higher order terms. Coupling this effective theory to gravity will be discussed in the next section.

[1] Strictly speaking this isn't quite true, because the diffeomorphism generated by $\xi^\mu = \partial^\mu\theta$ is enough to forbid a graviton mass term.

3.1 A Review of Ghost Condensation

We start with the "ghost condensate" model of Ref. [4]. This is an effective theory of a real scalar field ϕ with a global shift symmetry

$$\phi \to \phi + \lambda. \tag{3.1}$$

The effective theory is assumed to depend on a single scale M, which gives both the strength of self-interactions of the ϕ field and the cutoff of the effective theory. The novel feature of this sector is that it has a vacuum (in flat spacetime)

$$\langle \partial_\mu \phi \rangle = v_\mu, \tag{3.2}$$

where v_μ is a constant timelike vector. This spontaneously breaks Lorentz invariance by establishing a preferred time direction. We can then choose a Lorentz frame where $v_0 = c$, $v_i = 0$ for $i = 1, 2, 3$. In this frame we have

$$\langle \phi \rangle = ct. \tag{3.3}$$

It is convenient to take ϕ to have units of time, so that c is dimensionless and can be set to 1 by a rescaling of the time coordinate.

Expanding about the vacuum

$$\phi = \langle \phi \rangle + \pi, \tag{3.4}$$

we can construct a systematic effective field theory for the fluctuations π in derivatives simply by noting that

$$\begin{aligned} \partial_\mu \phi &= \delta_\mu{}^0 + \partial_\mu \pi, \\ \partial_\mu \partial_\nu \phi &= \partial_\mu \partial_\nu \pi. \end{aligned} \tag{3.5}$$

The constant part of $\partial_\mu \phi$ should be kept to all orders since there is no momentum suppression. On the other hand, terms with more than one derivative acting on ϕ only give rise to derivatives acting on the fluctuations, so there is a well-defined expansion at low energies and only the lowest order terms are important. The most general quadratic effective Lagrangian with at most four derivatives acting on π is

$$\begin{aligned} \mathcal{L}_{\text{ghost}} = M^4 P(X) + \tfrac{1}{2} M^2 \Big(& Q_1(X) (\Box \phi)^2 + Q_2(X) \partial^\mu \phi \partial^\nu \phi \partial_\mu \partial_\nu \phi \Box \phi \\ & + Q_3(X) (\partial^\mu \phi \partial^\nu \phi \partial_\mu \partial_\nu \phi)^2 \Big) + \cdots, \end{aligned} \tag{3.6}$$

where we have assumed a $\phi \to -\phi$ reflection symmetry, and

$$X = \partial^\mu \phi \partial_\mu \phi. \tag{3.7}$$

Expanding this Lagrangian about the vacuum using Eq. (3.5), the leading Lagrangian becomes

$$\mathcal{L}_{\text{ghost}} = \tfrac{1}{2}M^4\dot{\pi}^2 - \tfrac{1}{2}M^2\Big(\alpha(\vec{\nabla}^2\pi)^2 + \beta\ddot{\pi}\vec{\nabla}^2\pi - \gamma\ddot{\pi}^2\Big) + \mathcal{O}\left(\pi^3, (\partial^3\pi)^2\right). \tag{3.8}$$

Here we have rescaled the fields so that $P''(1) = \frac{1}{4}$, which fixes the coefficient of the $\dot{\pi}^2$ term, and used the fact that $P'(1) = 0$ in the vacuum so that there is no tadpole term. The coefficients of the other terms are given by

$$\begin{aligned} \alpha &= -Q_1(1), \\ \beta &= 2Q_1(1) + Q_2(1), \\ \gamma &= Q_1(1) + Q_2(1) + Q_3(1). \end{aligned} \tag{3.9}$$

The coefficients α, β, γ are independent as a result of the projection $\langle \partial_\mu \phi \rangle = \delta_\mu{}^0$. This is expected as Lorentz symmetry is broken. We can further rescale π and M to simplify this Lagrangian, but we will not make use of this freedom for now. We can see that there is no normal spatial kinetic term $(\vec{\nabla}\pi)^2$, which implies the special dispersion relation for π

$$\omega^2 = \alpha \frac{\vec{k}^4}{M^2} \tag{3.10}$$

to leading order in ω and k.

3.2 Gauged Ghost Condensation

We now couple the theory above to a gauge field A_μ by gauging the shift symmetry Eq. (3.1). That is, we promote λ to a local transformation and introduce a conventional $U(1)$ gauge field A_μ to make the Lagrangian invariant. It is convenient to define A_μ to have dimensions of mass, so the gauge transformations are

$$\begin{aligned} \delta A_\mu &= \partial_\mu \lambda, \\ \delta\phi &= -\frac{\lambda}{M}. \end{aligned} \tag{3.11}$$

The fields A_μ and ϕ must always appear in the gauge-invariant combination

$$\mathcal{A}_\mu = A_\mu + M\partial_\mu\phi = MD_\mu\phi, \tag{3.12}$$

which can also be thought of as the gauge convariant derivative of ϕ. The gauge-invariant order parameter that characterizes spontaneous Lorentz violation is now

$$\langle \mathcal{A}_0 \rangle = M. \tag{3.13}$$

The simplest Lagrangian coupling the vector to the ghost condensate is obtained by *weakly* gauging the $U(1)$ symmetry and assuming minimal coupling. In this theory, the leading terms in the Lagrangian consist of the ghost Lagrangian from Eq. (3.6) with the replacement $\partial_\mu \phi \to \mathcal{A}_\mu / M$ and the addition of an ordinary gauge kinetic term:

$$\mathcal{L}_{\text{gauged ghost}} = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{\text{ghost}}(\partial_\mu \phi \to \mathcal{A}_\mu / M). \tag{3.14}$$

Actually there is an ambiguity in this prescription since we can generalize $\partial_\mu \partial_\nu \phi$ to either $\partial_\mu \mathcal{A}_\nu$ or $\partial_\nu \mathcal{A}_\mu$ which are not equal. For example, $\partial_\mu \partial_\nu \phi \partial^\mu \partial^\nu \phi$ and $(\Box \phi)^2$ are equivalent through integration by parts, but $\partial_\mu \mathcal{A}_\nu \partial^\mu \mathcal{A}^\nu$ and $(\partial_\mu \mathcal{A}^\mu)^2$ are not. However, the difference only affects the speeds of the vector modes (and the tensor modes when coupled to gravity), but not the scalar mode. The general linearized Lagrangian around the vacuum $\langle \mathcal{A}_\mu \rangle = M {\delta_\mu}^0$ is then

$$\begin{aligned} \mathcal{L}_{\text{gauged ghost}} = & -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \tfrac{1}{2} M^2 (\mathcal{A}_0 - M)^2 \\ & - \tfrac{1}{2}\alpha_1 (\vec{\nabla} \cdot \vec{\mathcal{A}})^2 - \tfrac{1}{2}\alpha_2 (\partial_i \mathcal{A}_j)^2 + \tfrac{1}{2}\beta_1 (\partial_t \vec{\mathcal{A}})^2 - \tfrac{1}{2}\beta_2 (\vec{\nabla} \mathcal{A}_0)^2 \\ & + \beta_3 \partial_t \mathcal{A}_0 \vec{\nabla} \cdot \vec{\mathcal{A}} + \tfrac{1}{2}\gamma (\partial_t \mathcal{A}_0)^2 + \mathcal{O}\left((\partial^2 \mathcal{A})^2\right) + \mathcal{O}(\mathcal{A}^3). \end{aligned} \tag{3.15}$$

By the power counting of the ghost condensate effective theory, the coefficients $\alpha_i, \beta_i, \gamma$ are order 1, and give rise to general kinetic terms for the vector fields. While the $F^{\mu\nu} F_{\mu\nu}$ term does not generate any new kinetic terms, for $g \ll 1$ it gives the dominant kinetic contribution to the transverse modes. In the limit $g \to 0$, we recover the ghost condensate theory. For most part of the paper we will confine ourselves to the case where we can treat g as a small parameter. The results for the more general theory are not very different and we will comment on what happens for $g \gtrsim 1$ when necessary.

Eq. (3.15) contains a "mass" term for $\mathcal{A}_0$, so we can integrate out $\mathcal{A}_0$ if we are interested in energies and momenta below the scale M. To see this explicitly, we compute the dispersion relation for the scalar modes. We parameterize them by

$$A_0 = M + a, \qquad A_i = \partial_i \sigma, \qquad \pi. \tag{3.16}$$

Choosing "unitary gauge" $\pi \equiv 0$, the quadratic Lagrangian in momentum space is

$$\mathcal{L} = \frac{1}{2g^2} \begin{pmatrix} \sigma & a \end{pmatrix} K \begin{pmatrix} \sigma \\ a \end{pmatrix}, \tag{3.17}$$

where

$$K = \begin{pmatrix} (1 + g^2 \beta_1) \omega^2 \vec{k}^2 - g^2 (\alpha_1 + \alpha_2) \vec{k}^4 & i(1 - g^2 \beta_3) \omega \vec{k}^2 \\ -i(1 - g^2 \beta_3) \omega \vec{k}^2 & (1 - g^2 \beta_2) \vec{k}^2 + g^2 \gamma \omega^2 + g^2 M^2 \end{pmatrix} \tag{3.18}$$

The dispersion relation for the scalar modes is obtained by setting the determinant of the kinetic matrix to zero. We find two scalar modes, one with dispersion relation

$$\omega^2 = \alpha \left(g^2 \vec{k}^2 + \frac{\vec{k}^4}{M^2} \right) + \mathcal{O}(\vec{k}^6/M^4), \quad \alpha = \alpha_1 + \alpha_2, \tag{3.19}$$

and one with

$$\omega^2 = -\frac{M^2}{\gamma} + \mathcal{O}(\vec{k}^2), \tag{3.20}$$

where we have used $g^2 \ll 1$.

The first mode is massless and corresponds roughly to the longitudal mode σ. For $|\vec{k}| \ll gM$ we can neglect the quartic term in the dispersion relation, and such modes travel with speed $\sqrt{\alpha}g$ in the preferred frame. Modes with $|\vec{k}| \gg gM$ have a quartic dispersion relation, just as the Goldstone boson π in the (ungauged) ghost condensate. In this way, this theory approaches the theory of Ref. [4] as $g \to 0$. For $g \gg 1$, the $-(1/4g^2)F^{\mu\nu}F_{\mu\nu}$ term is suppressed relative to other kinetic terms, and the dispersion relation becomes $\omega^2 \approx \alpha\vec{k}^2/\beta_1$. This mode then travels with $\mathcal{O}(1)$ speed in the preferred frame, and the higher order terms such as $\vec{k}^4$ in the dispersion relation never become important within the regime of validity of the effective theory ($\omega, |\vec{k}| < M$).

The second mode has an energy gap of order M, and is therefore not a mode that is accurately described in the effective theory. We therefore write the effective theory below the scale M by integrating out the field A_0, which has an order-1 overlap with the massive mode. We can do this without fixing any gauge, and we obtain the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1 + g^2\beta_1}{2g^2}(\partial_t \vec{\mathcal{A}})^2 - \frac{1 + g^2\alpha_2}{4g^2} F_{ij}^2 - \tfrac{1}{2}\alpha(\vec{\nabla} \cdot \vec{\mathcal{A}})^2 + \mathcal{O}(1/M^2). \tag{3.21}$$

We have dropped $1/M^2$ terms which means that we have neglected the $\vec{k}^4$ term in the dispersion relation for the massless scalar mode. This is justified if we restrict attention to momenta satisfying $|\vec{k}| \ll gM$.[2]

What has happened is that the Goldstone mode π has been "eaten" by the gauge field A_μ and can be viewed as the longitudinal mode of the gauge field $\mathcal{A}_\mu$. However, unlike in the Lorentz-invariant Higgs mechanism, only $\mathcal{A}_0$ gets a mass term and the modes paramerized by $\vec{\mathcal{A}}$ remain massless. It is also easy to read from Eq. (3.21) that the velocity of the two transverse modes in the preferred frame is modified by $\Delta c_{\text{vector}} \approx (\alpha_2 - \beta_1)g^2/2$. Even without coupling to gravity, Eq. (3.21) is interesting in its own right, especially if we imagine other fields charged under the $U(1)$ gauge group. **(Expand the following discussion.)**

[2]The $\vec{k}^4$ correction can be obtained by keeping the $-(1/2g^4M^2)(\partial_t\vec{\nabla} \cdot \vec{\mathcal{A}})^2$ term, which is enhanced for small g.

In some sense, the Lorentz-violating Higgs mechanism removes "electric" interactions ($\mathcal{A}_0$) while preserving "magnetic" ones ($\mathcal{A}_i$). We will explore this interesting feature in the context of the Standard Model in Section 7.

4 Coupling the Gauged Ghost Condensate to Gravity

In ghost condensation, coupling to gravity induces an $M_{\rm Pl}$ suppressed $\vec{k}^2$ term in the dispersion relation for the Goldstone boson [4],

$$\omega^2 = \alpha \frac{\vec{k}^4}{M^2} \quad \rightarrow \quad \omega^2 = -\alpha \frac{M^2}{2M_{\rm Pl}^2}\vec{k}^2 + \alpha \frac{\vec{k}^4}{M^2}. \tag{4.1}$$

The negative sign of the $\vec{k}^2$ term implies an instability for long wavelength modes with $|\vec{k}| < M^2/\sqrt{2}M_{\rm Pl}$. The time scale for the instability is $t_c \sim M_{\rm Pl}^2/M^3$, and if we assume that there has been no manifestation of the instability over the age of the universe, this sets an upper bound for the energy scale of the Lorentz violation $M < 10$ MeV. Now in gauged ghost condensation, from Eq. (3.19) we see that for $\alpha > 0$ there is already a positive $\vec{k}^2$ piece in the dispersion relation proportional to g^2. For large enough g we expect that the linear instability will be absent when it is coupled to gravity, allowing us to push the scale of spontaneous Lorentz violation much higher.

4.1 Linear Theory

It is straightforward to couple the gauged ghost condensate to gravity since we started with a Lorentz-invariant formulation of the theory. Following the minimal coupling procedure, we simply replace the flat metric $\eta_{\mu\nu}$ by a dynamical metric $g_{\mu\nu}$ and replace ordinary derivatives ∂_μ by spacetime covariant derivatives ∇_μ .

Expanding about flat spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{4.2}$$

and following the steps above, the quadratic Lagrangian is

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\rm EH} &- \frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}M^2(\mathcal{A}_0 - \tfrac{1}{2}Mh_{00} - M)^2 \\ &- \tfrac{1}{2}\alpha_1(\vec{\nabla}\cdot\vec{\mathcal{A}})^2 - \tfrac{1}{2}\alpha_2(\nabla_i\mathcal{A}_j)^2 + \tfrac{1}{2}\beta_1(\nabla_t\vec{\mathcal{A}})^2 - \tfrac{1}{2}\beta_2(\vec{\nabla}\mathcal{A}_0)^2 \\ &+ \beta_3\nabla_t\mathcal{A}_0\vec{\nabla}\cdot\vec{\mathcal{A}} + \tfrac{1}{2}\gamma(\nabla_t\mathcal{A}_0)^2 + \cdots \end{aligned} \tag{4.3}$$

where $\mathcal{L}_{\rm EH}$ is the Einstein-Hilbert Lagrangian. The weak gauging of gravity affects the dispersion relation for the scalar modes only by terms suppressed by $1/M_{\rm Pl}$, so we can still integrate out the massive mode A_0 as before. The leading effective Lagrangian for small g is then

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm EH} + \frac{1}{2g'^2}(\nabla_t \vec{\mathcal{A}})^2 - \frac{1}{4g''^2}F_{ij}^2 - \tfrac{1}{2}\alpha(\vec{\nabla}\cdot\vec{\mathcal{A}})^2 + \cdots, \tag{4.4}$$

where

$$\frac{1}{g'^2} = \frac{1}{g^2} + \beta_1, \qquad \frac{1}{g''^2} = \frac{1}{g^2} + \alpha_2. \tag{4.5}$$

In this expression,

$$\nabla_t \mathcal{A}_i = \partial_t \mathcal{A}_i - \frac{M}{\sqrt{2}M_{\rm Pl}}\partial_i \Phi^c, \tag{4.6}$$

where $\Phi^c = \sqrt{2}M_{\rm Pl}\Phi = M_{\rm Pl}h_{00}/\sqrt{2}$ is the canonically normalized Newtonian potential. Note that the $(\nabla_t \vec{\mathcal{A}})^2$ term contains mixing between $\vec{\mathcal{A}}$ and $h_{\mu\nu}$. It is in this sense that the theory is a modification of gravity. Because this is kinetic mixing, it gives a much less dramatic modification of gravity at long distances compared to mixing through mass terms (with no derivatives), as is the case in the original ghost condensate.

Note that the effective Lagrangian in Eq. (4.4) is fully invariant under gauge transformations and diffeomorphisms:

$$\begin{aligned}
\delta\pi &= -\xi_0 - \frac{\lambda}{M}, \\
\delta A_i &= \partial_i \lambda, \\
\delta h_{\mu\nu} &= -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu.
\end{aligned} \tag{4.7}$$

In fact, the quadratic terms in Eq. (4.4) are the most general ones invariant under these symmetries, and armed with the foresight to integrate out A_0, we could have used Eq. (4.7) as the starting point for our analysis. Indeed, this is exactly what we suggested in Section 2 when we posited the existence of a χ_i field. The advantage of thinking of Eq. (4.4) as coming from gauged ghost condensation is that the weak gauging procedure explains why g' and g'' are nearly degenerate.

The modified dispersion relation for the scalar excitation can be easily derived. Parametrizing $\mathcal{A}_i = \partial_i \sigma$ and choosing Newtonian gauge, we have

$$\mathcal{L} \supset \frac{1}{2}\begin{pmatrix} \sigma/g & \Phi^c \end{pmatrix} \begin{pmatrix} \omega^2\vec{k}^2 - \alpha g^2 \vec{k}^4 & -i\epsilon\omega\vec{k}^2 \\ i\epsilon\omega\vec{k}^2 & -\vec{k}^2(1-\epsilon^2) \end{pmatrix} \begin{pmatrix} \sigma/g \\ \Phi^c \end{pmatrix}, \tag{4.8}$$

where again

$$\epsilon = \frac{M}{gM_{\rm Pl}\sqrt{2}}, \tag{4.9}$$

and we have taken the small g limit and dropped the subleading terms suppressed by g^2. (The second term in the (1,1) entry gives the leading contribution in $\vec{k}^2$ in the dispersion relation so it needs to be kept.) By setting the determinant of the kinetic matrix to zero, we obtain

$$\omega^2 = \alpha(g^2 - g_c^2)\,\vec{k}^2, \tag{4.10}$$

where $g_c = \epsilon g = M/(\sqrt{2}M_{\rm Pl})$. If we had kept the leading contribution at $\mathcal{O}(\vec{k}^4/M^2)$, the dispersion relation would have been

$$\omega^2 = \alpha(g^2 - g_c^2)\,\vec{k}^2 + \alpha\frac{\vec{k}^4}{M^2}. \tag{4.11}$$

We can see that if $g > g_c$, this is a healthy dispersion relation and there is no instability.

Note that the dispersion relation of the tensor modes is also modified because $(\nabla_i \mathcal{A}_j)^2$ contains a term $(\mathcal{A}_0/2)\partial_t h_{ij}$ which contributes to the time derivative of h_{ij} after $\mathcal{A}_0$ takes a vev. As a result, the speed of the usual gravitational waves is modified by $\mathcal{O}(\alpha_2 M^2/M_{\rm Pl}^2)$. If the speed of the gravitational wave is smaller than the speeds of ordinary particles, there is a strong constraint from cosmic rays. **(cite Moore and Nelson)**

4.2 Modification of Gravity and Observational Constraints

Because of the mixing between the Goldstone boson and the metric tensor, the gravitational potential is modified. In the static limit, $\omega \to 0$, the only modification is a simple renormalization of Newton's constant,

$$G_N = \frac{G_N^0}{1-\epsilon^2} = \frac{1}{8\pi M_{\rm Pl}^2(1-\epsilon^2)}, \tag{4.12}$$

as one can see from Eq. (4.8). By itself, this does not lead to any measurable effects. However, this static limit only applies to matter sources at rest with respect to the local ether rest frame. There will be non-trivial effects if the source is moving relative to the ether rest frame as expected in the real world. For a source of mass $M_\odot$ moving with velocity $\vec{v}$ relative to the ether rest frame, the stress-energy tensor is

$$T_{00} = \rho = M_\odot \delta^{(3)}(\vec{x} - \vec{v}t), \tag{4.13}$$

and T_{0i} and T_{ij} are down by powers of $v = |\vec{v}|$ and can be ignored for $v \ll 1$. The gravitational potential can be easily obtained through the $\Phi\Phi$ propagator by inverting the kinetic matrix in Eq. (4.8),

$$\langle \Phi\Phi \rangle = -\frac{1}{2M_{\rm Pl}^2}\frac{1}{\vec{k}^2}\left(1 - \frac{\alpha g^2 \epsilon^2 \vec{k}^2}{\omega^2 - \alpha g^2(1-\epsilon^2)\vec{k}^2}\right). \tag{4.14}$$

The Fourier transform of the source is

$$\tilde{\rho}(\omega, \vec{k}) = 2\pi M_\odot \delta(\omega - \vec{k}\cdot\vec{v}). \tag{4.15}$$

The Newtonian potential is obtained by performing the inverse Fourier transform of the $\langle\Phi\Phi\rangle$ propagator after substituting ω by $\vec{k}\cdot\vec{v}$. Depending on the source velocity in the preferred frame v relative to the velocity of the scalar Goldstone mode $c_s \equiv \sqrt{\alpha(g^2 - g_c^2)}$, we get very different results. It is important to distinguish the two different cases

For $v < c_s$, we can expand in powers of v/c_s and obtain the angular-dependent gravitational potential

$$V(r,\theta) = -\frac{G_N^0 M_\odot}{r}\left[1 + \frac{\epsilon^2}{1-\epsilon^2}\left(1 + \frac{v^2}{2c_s^2}\sin^2\theta + \mathcal{O}(v^4/c_s^4)\right)\right], \tag{4.16}$$

where $\cos\theta = \hat{r}\cdot\hat{v}$ is the angle measured from the direction of the source moving with respect to the ether.

In the parametrized post-Newtonian (PPN) formalism this velocity effect with respect to the preferred frame corresponds to the PPN parameter $\alpha_2^{\rm PPN}$,

$$\alpha_2^{\rm PPN} = \frac{\epsilon^2}{(1-\epsilon^2)c_s^2} = \frac{M^2}{2\alpha g^4(1-\epsilon^2)^2 M_{\rm Pl}^2}. \tag{4.17}$$

The observational bound on $\alpha_2^{\rm PPN}$ is

$$\alpha_2^{\rm PPN} < 4\times 10^{-7} \tag{4.18}$$

from the alignment of the solar spin axis and its ecliptic [25, 26]. This provides the strongest bound on the scale of Lorentz symmetry breaking in this case.[3] For αg^4 close to 1 we have $M \lesssim 10^{15}\sqrt{\alpha}g^2$ GeV, and the constraint is stronger for smaller g. A more systematic derivation of the modification of gravity and discussion of other PPN parameters can be found in the Appendices. In particular, we find that $\alpha_2^{\rm PPN}$ is the only modification of General Relativity at post-Newtonian order, showing that gauged ghost condensation is a very mild modification of gravity in this case. Our analysis also exhibits the applicable range of the PPN formalism in this theory. *It is not enough just for v to be small because the expansion parameter is actually v/c_s.* It requires $v < c_s$ and the mixing parameter ϵ to be small. The above result holds for any theory where Lorentz invariance is broken by the vev of a vector field (such as Ref. [**?**]).

On the other hand, if $v > c_s$, the PPN expansion breaks down. We expect that there will be Čerenkov radiation of the Goldstone field. By casuality, all the modifications of gravity

[3]There is also a bound from the rate of the cosmic expansion during the Big Bang necleosynthesis due to the different effective Newton's constant for the cosmological evolution [28], but it is much weaker.

happen only in a cone with an angle $\theta = \sin^{-1}(c_s/v)$ behind the source.[4] For $v \gg c_s$, the cone is narrow and we may not see modifications of gravity in the solar system where the most stringent bounds come from, if the cone lies outside the ecliptic plane. (Of course, there will be some anomalous acceleration if an astrophysical object happens to move into the cone of shadow of a gravitational source.) The most intereting effects in this case probably come from the energy loss due to the Čerenkov radiation. Because the coupling of the Goldstone mode to gravity is proportional to M, the Čerenkov radiation due to gravity scales as M^2. The ability to raise the Lorentz symmetry breaking scale M in gauged ghost condensation makes this effect interesting. In fact, it provides a significant bound on M. In constrast, in ungauged ghost condensation, the bound $M \lesssim 10$ MeV renders this effect totally irrelevant. However, to discuss the Čerenkov radiation from moving stars or planets, we needs to first understand the nonlinear effects and where they become important, so we postpone the discussion of Čerenkov radiation to Sec. 6 after we discuss the nonlinear effects.

5 Nonlinear Effects

Up to now we have confined ourselves to the linearized analysis. However, in (ungauged) ghost condensation, we learned that nonlinear effects are important for any realistic sources in the universe, such as stars and galaxies [29, 30]. Let us briefly review it here. The leading Lagrangian for ghost condensation is

$$\mathcal{L} = \frac{M^4}{8}(X-1)^2 - \frac{\alpha M^2}{2}(\Box\phi)^2, \tag{5.1}$$

where $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Expanding $X-1$ in the presence of a gravitational potential Φ gives

$$\frac{1}{2}(X-1) = \dot{\pi} - \Phi - \frac{1}{2}(\vec{\nabla}\pi)^2 + \cdots, \tag{5.2}$$

where the $(\vec{\nabla}\pi)^2$ term represents the leading nonlinear effects. It can becomes more important than the linear $(\Box\phi)^2$ term near a source. One can show that if one ignores the $(\Box\phi)^2$ term, the solution of $X = 1$ is simply that ϕ moves along the geodesics. The time scale for the nonlinear effects near a source is therefore given by the infall time (or orbit time),

$$t_{\text{infall}} \sim \frac{r}{\sqrt{\Phi}}, \tag{5.3}$$

where r is the distance to the source. The nonlinear effects are important when the infall time is shorter compared with the time scale of the linear propagation,

$$\frac{r}{\sqrt{\Phi}} < t_{\text{linear}} \sim \frac{Mr^2}{\sqrt{\alpha}} \quad \Rightarrow \quad \Phi > \frac{\alpha}{M^2 r^2}, \tag{5.4}$$

[4]This is true for distance larger than $(gM)^{-1}$. For $gM < |\vec{k}| < M$, the $\vec{k}^4$ term in the dispersion relation becomes more important than the $\vec{k}^2$ term, and the scalar velocity is enhanced and becomes $\vec{k}$ dependent, $c_s \sim \sqrt{\alpha}|\vec{k}|/M$. The distance scale where this is relevant is very small if M is high and g is not too small.

which is easily satified near a star for any interesting range of M. The nonlinear evolutions generate caustics. The regions with negative value of $X-1$ will shrink in size and grow in amplitude, and eventually go out the of valid regime of the effective theory. Although it does not necessarily imply diasters since the energy involved in these regions is quite small, it can cause some uneasiness that the effective theory breaks down and we do not have control over what happens in these regions.

In gauge ghost condensation, we also expect that nonlinear effects become important when $t_{\text{infall}} < t_{\text{linear}}$, except that t_{linear} is now given by

$$t_{\text{linear}} \sim c_s^{-1} r. \tag{5.5}$$

This can be derived as follows. By expanding $X-1$ to include the leading nonlinear interaction,

$$\frac{1}{2}(X-1) = \frac{\mathcal{A}_0}{M} - \Phi - \frac{1}{2M^2}\vec{\mathcal{A}}^2 + \cdots, \tag{5.6}$$

the leading Lagrangian is given by

$$\mathcal{L} = \frac{M^2}{2}\left(\mathcal{A}_0 - M\Phi - \frac{1}{2M}\vec{\mathcal{A}}^2\right)^2 - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{2}(\vec{\nabla}\cdot\vec{\mathcal{A}})^2. \tag{5.7}$$

Integrating out the massive $\mathcal{A}_0$ mode,

$$\mathcal{A}_0 = M\Phi + \frac{1}{2M}\vec{\mathcal{A}}^2, \tag{5.8}$$

we obtain

$$\frac{1}{2g^2}F_{0i}^2 = \frac{1}{2g^2}\left(\partial_0\vec{\mathcal{A}} - M\vec{\nabla}\Phi - \frac{\vec{\nabla}\vec{\mathcal{A}}^2}{2M}\right)^2 \tag{5.9}$$

Nonlinear effects become important when

$$\frac{1}{2g^2}\left(\frac{\vec{\nabla}\vec{\mathcal{A}}^2}{2M}\right)^2 > \frac{\alpha}{2}(\vec{\nabla}\cdot\vec{\mathcal{A}})^2. \tag{5.10}$$

If we take $\vec{\mathcal{A}}^2 \sim M^2\Phi$ and $\vec{\nabla} \sim 1/r$, the condition becomes

$$\sqrt{\Phi} > g\sqrt{\alpha} \ \ (\sim c_s,\ \text{for } g \gg g_c), \tag{5.11}$$

which is the same as $t_{\text{infall}} < t_{\text{linear}}$. If the velocity of the Goldstone mode is small, $g\sqrt{\alpha} \ll 1$, nonlinear effects can be important near a source. In those regions, $\mathcal{A}^\mu$ roughly follows charged particle lines in the background of the gravitational potential and $U(1)$ field. One expects that caustics will be less dangerous in this case because the charge density built-up will prevent the unbounded growth of the singular regions. Indeed, in the numerical simulations (which will be described in more details later) we find that in the presence of a gravitational potential, a bounce occurs at time $\sim t_{\text{infall}}$ and the gauged ghost condensate is smoothed out in region after the bounce, without any singular behavior. In particular, the nonlinear effects will align the gauged ghost condensate, *i.e.,* the local preferred frame, with the gravitational source. Therefore, for a star or other gravitational sources moving with a velocity v relative to the preferred frame will drag a nearby region of the gauged ghost condensate with it and establish a local preferred frame different from the global one. The size of the region is the region where the nonlinear effects are important and the orbital velocity around the star is bigger than the velocity of the star v, so that the nonlinear dynamics has enough time to create the local preferred frame. For the Sun, if we assume that it moves with a velocity $v \sim 10^{-3}$, it can drag a region only about its own size if $c_s \lesssim 10^{-3}$, For larger c_s it will be entirely in the linear regime. Therefore, we expect that the bounds derived in the previous section based on linear analysis should be valid.

5.1 Nonlinear dynamics

We have seen that the sound velocity is real and that the gauged ghost condensation is indeed stable against linear perturbations if the gauge coupling is large enough. In this section we ask the question "Is there any instabilities which do not show up in the linear analysis but which appears in the nonlinear regime?" To answer this question we shall perform numerical simulations. In the following analysis we shall work in fixed spacetimes since gravitational backreaction is always suppressed by M^2/M_{Pl}^2. To make our arguments independent of heavy modes above the cutoff M, we shall use the effective Lagrangian

$$\mathcal{L} = \frac{1}{2g^2}\left(\partial_0\vec{\mathcal{A}} - M\vec{\nabla}\Phi - \frac{\vec{\nabla}\vec{\mathcal{A}}^2}{2M}\right)^2 - \frac{1}{4g^2}F_{ij}F^{ij} - \frac{\alpha}{2}(\vec{\nabla}\cdot\vec{\mathcal{A}})^2, \tag{5.12}$$

which has been obtained by integrating $\mathcal{A}_0$ out.

We consider a fixed flat spacetime background with the metric

$$ds^2 = L_{phys}^2\left[dt^2 - dr^2 - r^2 d\Omega_s^2 - \sum_{p=1}^{2-s}(dx_p)^2\right], \tag{5.13}$$

where $d\Omega_s^2$ is zero for $s=0$ (planar symmetry), $d\theta^2$ for $s=1$ (cylindrical symmetry) and $d\theta^2 + \sin^2\theta d\varphi^2$ for $s=2$ (spherical symmetry). For the symmetric configuration

$$\mathcal{A}_i dx^i = ML_{phys}A_r(t,r)dr, \tag{5.14}$$

there is no "magnetic field" ($F_{ij}=0$) and the equation of motion is written as

$$\partial_t E_r - \frac{A_r}{r^s}\partial_r(r^s E_r) + j_r = 0, \tag{5.15}$$

where

$$E_r \equiv \partial_t A_r - \partial_r \Phi - A_r \partial_r A_r, \quad j_r \equiv -\alpha g^2 \partial_r \left[\frac{1}{r^s} \partial_r (r^s A_r) \right]. \tag{5.16}$$

The equation (5.15) can be interpreted as an analogue of the Ampere's law

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \tag{5.17}$$

with $\mathbf{B} = 0$. Indeed, (5.15) is rewritten as

$$\mathcal{A}^\mu \partial_\mu (r^s E_r) = -r^s j_r, \tag{5.18}$$

and states that the derivative of the "flux" $r^s E_r$ with respect to the proper time along the 4-vector $\mathcal{A}^\mu \simeq (1, -A_r, 0, 0, 0)$ is supplied by minus the "current density" j_r multiplied by the "area" r^s. In particular, when the "current density" is absent, the "flux" conserves. On the other hand, we do not have an analogue of the Gauss's law since $\mathcal{A}_0$ is heavy and we have integrated it out. The conservation of the "flux" based on the analogue of Ampere's law will be useful to understand the numerical results.

The 4-vector $\mathcal{A}^\mu$ describes the behavior of the gauged ghost condensate. Hence, it is useful to consider a congruence of timelike curves generated by $\mathcal{A}^\mu$. The congruence defines the rest frame of the gauged ghost condensate: when $A_r < 0$, the rest frame of the gauged ghost condensate is expanding; one the other hand, when $A_r > 0$, the rest frame is contracting. In this sense A_r corresponds to, so to speak, a "focusing velocity". The equation governing the "focusing velocity" A_r is obtained by rewriting the definition of E_r as

$$\mathcal{A}^\mu \partial_\mu A_r = E_r + \partial_r \Phi. \tag{5.19}$$

This equation can be understood as Newton's second law: A_r is a "focusing velocity" and thus $\mathcal{A}^\mu \partial_\mu A_r$ is an acceleration, E_r is "electric force", and $\partial_r \Phi$ is of course gravitational force. This implies that a region with positive $E_r + \partial_r \Phi$ attracts the gauged ghost condensate and a region with negative $E_r + \partial_r \Phi$ repels the gauged ghost condensate. Note that the external gravitational force is always attractive, $\partial_r \Phi > 0$, while the "electric force" E_r can be either attractive or repulsive. In the following we show both analytically and numerically that in a contracting phase, the "electric force" becomes repulsive and strong enough to stop the contracting phase and bounces the would-be caustics.

Suppose that a dilute gas of dust quickly collapses to form a compact object at $t = 0$. Before the formation of the compact object, the natural initial condition is $A_r = E_r = 0$. By the formation, gravity becomes attractive, i.e. $\partial_r \Phi > 0$, but A_r and E_r are still zero. Now let us consider the evolution after that. For this purpose it is convenient to follow the evolution along each timelike curve generated by the 4-vector $\mathcal{A}^\mu$ and to introduce a new radial coordinate r_0 as the value of r at $t = 0$. Note that r decreases (or increases, respectively) as t increases with r_0 fixed if the rest frame of the gauged ghost condensate is contracting (or expanding).

The leading behavior of A_r near $r = 0$ is linear in r, but A_r inevitably has a non-vanishing second derivative off $r = 0$ unless $A_r = 0$ everywhere since $A_r = 0$ at large r. If the small r_0 region continued to be attractive, namely if the r.h.s. of (5.19) continued to be positive for small r_0, then $\partial_r A_r$ would eventually become positive and large in the small r_0 region. Since $A_r = 0$ for large r, $\partial_r^2 A_r$ for small r_0 would become negative when $\partial_r A_r$ becomes large enough. This implies that j_r defined by (5.16) should eventually become positive in the small r_0 region, provided that $\alpha g^2 > 0$.

Since j_r becomes positive in the central region with $E_r + \partial_r \Phi > 0$, the equation of motion (5.18) implies that $r^s E_r$ should decrease. Since we start from $E_r = 0$, this means that E_r should become negative.

With this observation and by the following reason, we can show that for $s = 2$ and $s = 1$, the would-be caustics, or the contracting region, should bounce. What is important here is the already stated fact that eq. (5.19) can be understood as Newton's second law. Because of this fact and the equation (5.18), a region with negative E_r tends to diffuse itself. Indeed, it would cost an infinite amount of work to compress a finite-volume, negative $r^2 E_r$ region to a point since the equation (5.18) with $s = 2$ implies that the "electric force" E_r is roughly proportional to $1/r^2$. Therefore, a caustics cannot occur at a point. How about a caustic string? This is also impossible by the same reason: it would again cost an infinite amount of work to compress a cylindrical region with negative rE_r to a string with infinitesimal thickness since the "electric force" E_r is roughly proportional to $1/r$.

On the other hand, it is in principle possible to create a caustic plane ($s = 0$) since it costs only a finite amount of work to compress a thick layer to an infinitesimal thin layer if perfect plane symmetry is assumed. However, this situation is unlikely to happen in generic situations since the perfect plane symmetry is too ideal an assumption. Indeed, small fluctuations on top of the plane symmetric collapse should grow and the layer should be broken into pieces before a planar caustics occurs. After the fragmentation, a lower-dimensional caustics does not occur since, as discussed above, for codimension 2 or higher it would cost infinite amount of work to compress a finite-volume, negative $r^s E_r$ ($s = 1, 2$) region to an infinitesimal volume or thickness.

Note that the large r limit of the $s = 2$ and $s = 1$ cases locally correspond to the plane symmetric case ($s = 0$), provided that the frame is properly boosted to the local comoving frame. Hence, it is in principle possible that a shell-crossing type caustics may happen for $s = 2$ and $s = 1$ if $A_r < 0$ and $\partial_r A_r > 0$ at large r. However, this type of caustics is also unlikely to happen in generic situations since inhomogeneous perturbations should grow on the locally contracting codimension-1 layer and the layer should be broken into pieces. After the fragmentation, again, a lower-dimensional (or shell-focusing type) caustics in the local comoving frame does not occur.

In summary there should be no instability except for the perfectly plane symmetric case. In the following we shall show by numerical simulations that this is indeed the case. As stated above, deviation from the perfect plane symmetry should naturally grow and, thus, we expect that nonlinear dynamics does not introduce any instabilities in generic situations.

To reach this result, it was important to note that E_r is inevitably generated. Note that this effect is nonlinear. In the linearized level E_r is a transverse mode (or a vector mode) and, thus, cannot be generated from a longitudinal mode (or a scalar mode). This is not the case in the nonlinear level and, as we have seen, a negative E_r is generated during the formation of the would-be caustics.

Before showing numerical results, it must be noted that the equation of motion permits exact scaling solutions. Indeed, the configuration

$$A_r = \frac{N_s r}{t_c - t}, \quad E_r = \frac{N_s(N_s-1)r}{(t_c-t)^2}, \tag{5.20}$$

satisfies the above equation of motion, where $N_s = 1, 2/(s+1)$ and t_c is an arbitrary constant. For these solutions the α term, or the "current density" j_r, vanishes and, thus, does not prevent a singularity from occurring at $t = t_c$. Note that the existence of these solutions by itself does not imply non-linear instabilities since the realistic boundary condition $A_r|_{r=\infty} = E_r|_{r=\infty} = 0$ removes these solutions. Nonetheless, they can in principle approximate some transient behavior near the center of symmetry.

Hence, just to be sure about the nonlinear stability, we would like to show that for $s = 1$ and $s = 2$, the scaling solutions are dynamically avoided.

Suppose that we start with $A_r = 0$ at large r and imagine that one of the scaling solutions develops near $r = 0$. In the development of the scaling solution ($A_r \simeq r/(t_c - t)$) near $r = 0$, $\partial_r A_r$ becomes large near $r = 0$ and propagates from the center to the outside. This inevitably generates the second derivative of A_r, which means that j_r becomes non-zero. The positivity of $\partial_r A_r$ for the scaling solution and $A_r = 0$ at large r imply that j_r defined by (5.16) should tend to become positive, provided that $\alpha g^2 > 0$.

Since j_r becomes positive during the formation of the would-be caustics, the equation of motion (5.18) implies that E_r should decrease from what we would expect from the scaling solutions. For $s = 1$ and $s = 2$, since E_r would be zero or negative for the scaling solutions, E_r should be negative during the development of the scaling solutions.

With the negativity of E_r, by repeating the above discussion, we can show that the collapsing scaling solution should bounce for $s = 2$ and $s = 1$. What is important here is

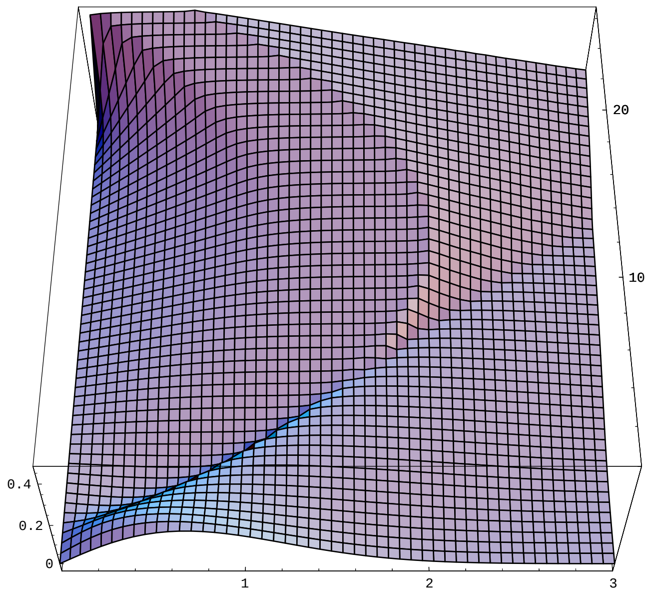

Fig. 1. A result of numerical simulation in plane symmetry ($s = 0$). The variable A_r is plotted as a function of r and t.

again the fact that E_r is roughly proportional to $1/r^s$ and that eq. (5.19) can be understood as Newton's second law.

Now it is the time to show numerical results. To begin with, let us consider the plane symmetric case ($s = 0$) without the external gravitational potential ($\Phi = 0$). For simplicity we assume the symmetry under reflection $r \to -r$. In this case A_r and E_r are odd functions of r. As the initial condition at $t = 0$ we set

$$A_r|_{t=0} = 2\Omega r e^{-r^2}, \quad E_r|_{t=0} = 0, \tag{5.21}$$

where Ω is a positive constant. As shown in Fig. 1, for some values of αg^2 and Ω, numerical simulations show that a would-be caustics develops at $t \sim 1/\Omega$ but it bounces; and that another caustics forms long after the would-be caustics, and it does not bounce until the system exits the regime of validity of the effective field theory. For other values of αg^2 and Ω, even the first caustics at $t \sim 1/\Omega$ does not bounce until the system exits the regime of validity of the effective field theory.

The occurrence of caustics in the perfect plane symmetric case is what we expected and must not be considered as a serious problem. Instead, it should be interpreted just as an indication that the system near the would-be caustics should deviate from the perfect plane symmetry. Small fluctuations on top of the plane symmetric collapse should grow and the layer with negative E_r should be broken into pieces before reaching a real singularity. After the fragmentation we should consider configurations with more than one codimensions, which we shall analyze now with cylindrical ($s = 1$) and spherical ($s = 2$) symmetries.

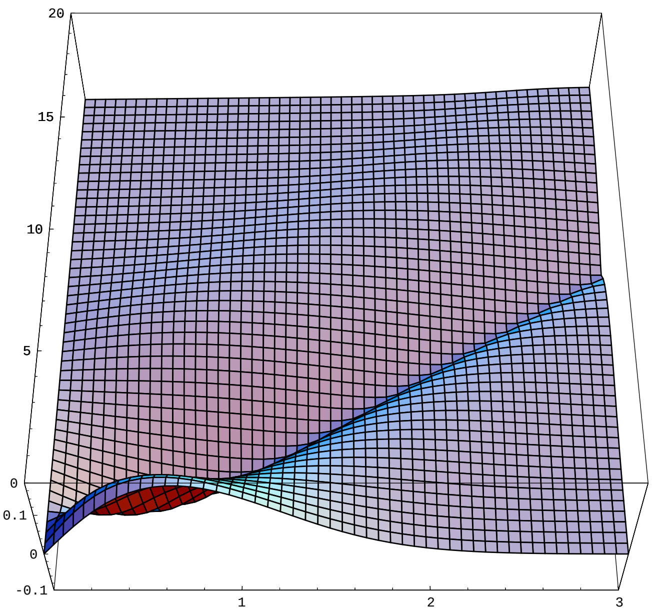

Fig. 2. A result of numerical simulation in cylindrial symmetry ($s = 1$). The variable A_r is plotted as a function of r and t.

Fig. 2 and Fig. 3 show results of numerical simulation in cylindrical and spherical cases with $\Phi = 0$, respectively. It is evident that the lump of excitations diffuses and the system asymptotically approaches the trivial solution $A_r = E_r = 0$. These results confirm our expectation that caustics with codimensions more than one cannot occur.

So far, we have been working in the absence of gravity. Let us now consider effects of a gravitational potential generated by external matter source. We shall still neglect gravity generated by excitations of the gauged ghost condensation and work in a fixed spacetime background, but the spacetime is now curved. For simplicity we assume spherical symmetry ($s = 2$). In the presence of the external gravitational potential, a numerical result is shown in Figs. 4 and 5. In the final stationary configuration, the repulsive "electric force" E_r cancels the attractive gravitational force $\partial_r \Phi$. Namely, $A_r \to 0$, $E_r \to -\partial_r \Phi$.

6 Čerenkov radiation from gravitating sources

For a gravitational source moving faster than the sound velocity of the ether, $v < c_s$, the ether Čerenkov radiation will be emitted. As we will see, the energy loss is proportional to M^2. This effect is totally unobservable for the ungauged ghost condensation where $M \lesssim 10$ MeV. In gauged ghost condensation M can be much higher, which may allow interesting phenomena and constraints from this effect.

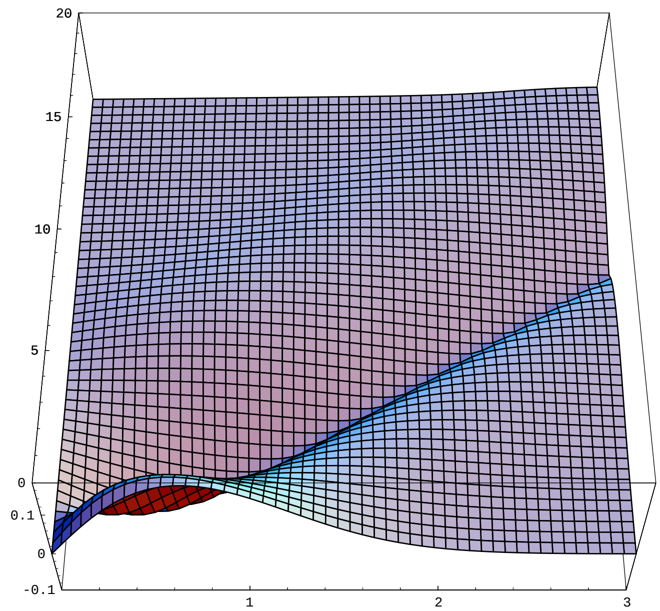

Fig. 3. A result of numerical simulation in spherical symmetry ($s = 2$). The variable A_r is plotted as a function of r and t.

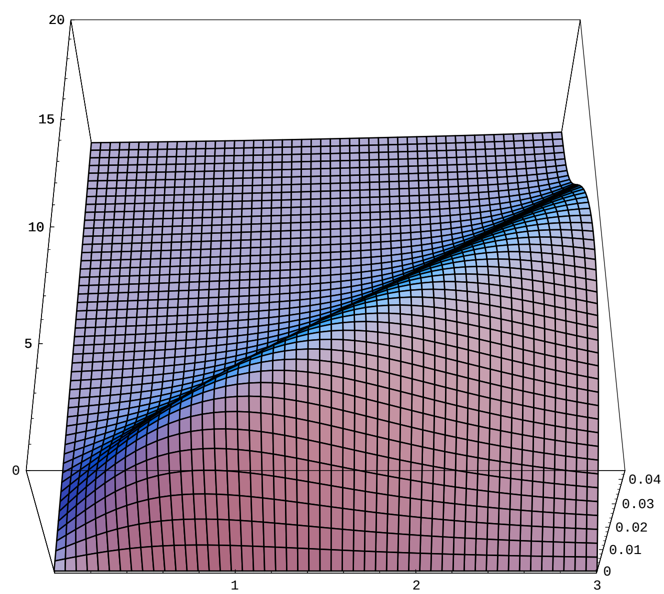

Fig. 4. A result of numerical simulation in spherical symmetry ($s = 2$) with an external gravitational potential. The variable A_r is plotted as a function of r and t.

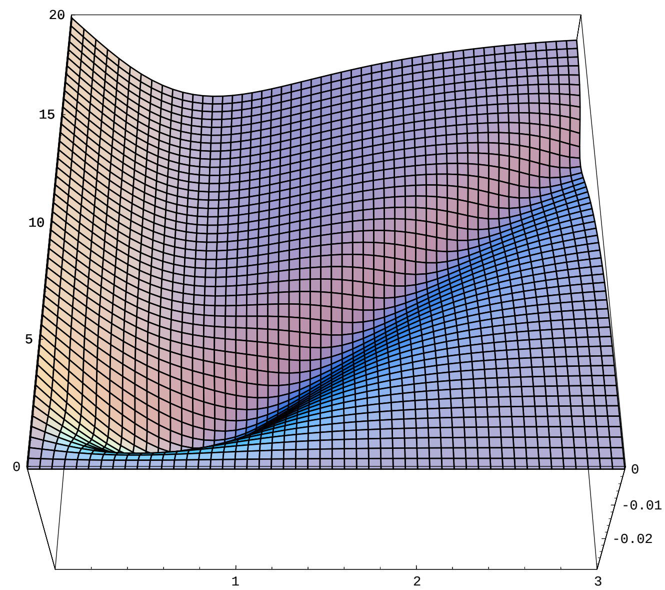

Fig. 5. A result of numerical simulation in spherical symmetry ($s = 2$) with an external gravitational potential. The variable E_r is plotted as a function of r and t.

Consider a non-relativistic source

$$\mathcal{L}_{\mathcal{S}} = -\frac{1}{\sqrt{2}M_{\rm Pl}}\rho\,\Phi^c. \tag{6.1}$$

As shown in the Appendix ?, the energy loss rate can be calculated as

$$\frac{dE_S}{dt} = \int d^3x \frac{1}{\sqrt{2}M_{\rm Pl}}\rho(x,t)\dot{\Phi}^c(x,t). \tag{6.2}$$

At the linearized level,

$$\tilde{\Phi}^c(\omega,k) = \left[-\frac{1}{k^2} + \frac{\alpha M^2}{2M_{\rm Pl}^2}\,\frac{1}{\omega^2 - c_s^2 k^2}\right]\frac{\tilde{\rho}(\omega,k)}{\sqrt{2}M_{\rm Pl}}, \tag{6.3}$$

where we have ignored the k^4 piece in the denominator of the second term in the bracket because it is never important for the large astrophysical bodies we consider here.

For a star of mass M_0 moving on a straight line with velocity $\vec{v}$, we have

$$\tilde{\rho}(\omega,k) = 2\pi M_0 \delta(\omega - \vec{k}\cdot\vec{v})f(k), \tag{6.4}$$

where $f(k)$ is the dimensionless form factor given by the Fourier transform of the mass density distribution. A uniform-density sphere of radius R_0 gives

$$f(k) = \frac{3(\sin kR_0 - kR_0\cos kR_0)}{(kR_0)^3}. \tag{6.5}$$

Substitute Eq. (6.4) into Eqs. (6.2) and (6.3), we have

$$\frac{dE_S}{dt} = \frac{M_0^2}{2M_{\rm Pl}^2}\int \frac{d^3k}{(2\pi)^3}(-i\vec{k}\cdot\vec{v})\left[-\frac{1}{k^2} + \frac{\alpha M^2}{2M_{\rm Pl}^2}\frac{1}{(\vec{k}\cdot\vec{v}^2 + i\varepsilon)^2 - c_s^2 k^2}\right]|f(k)|^2, \tag{6.6}$$

where we have used the $i\varepsilon$ prescription for the retarded Green's function. Note that the integral in only nonzero due to the poles of the second term, *i.e.*, when the propagator is on shell. This occurs only for $v > c_s$, and we obtain

$$\frac{dE_S}{dt} = -\frac{\alpha M_0^2 M^2}{16\pi M_{\rm Pl}^4 v}\int_0^\infty k\, dk|f(k)^2|. \tag{6.7}$$

For a source of size R_0, we expect

$$\int_0^\infty k\, dk|f(k)^2| \sim \frac{1}{R_0^2}. \tag{6.8}$$

However, as we discussed in the previous section, nonlinear effects can be important near a source. Iin particular, within a distance $r < R_{\rm nonlinear} \approx M_0/(M_{\rm Pl}^2 c_s^2)$ to the source, nonlinear effects become important, and within $r < R_{\rm dragged} \approx M_0/(M_{\rm Pl}^2 v^2)$, ether will be dragged by the source. We do not expect Čerenkov radiation to be emitted in the dragged region, so if $R_{\rm dragged} > R_0$, the integral (6.7) should be cut off by $R_{\rm dragged}^{-1}$. In the nonlinear region, $R_{\rm dragged} < r < R_{\rm nonlinear}$, the linear calculation of the Čerenkov radiation strictly speaking is not valid, but an estimate of the energy loss in the nonlinear region gives the same order and parameter dependence as the linear result. **(cite Markus)** We will parametrize this complication into an effective form factor and we expect

$$\int_0^\infty k\, dk|f_{\rm eff}(k)^2| \sim \frac{\kappa}{{\rm Max}(R_{\rm dragged},\, R_0)^2} \tag{6.9}$$

with $\kappa \lesssim \mathcal{O}(1)$. We already saw for the Sun, $R_{\rm dragged} \sim 2R_\odot$ if $v \sim 10^{-3}$. Other planets in the solar system all have $R_{\rm dragged} < R_0$, so we can simply use the radii as the cutoffs in these formulae.

The energy loss induces an anomalous acceleration in the direction of the ether wind,

$$a_{\rm anom} = \frac{dE_S/dt}{M_0 v} \approx \frac{\alpha\kappa M_0 M^2}{16\pi M_{\rm Pl}^4 v^2 R_0^2}. \tag{6.10}$$

Because it is proportional to M_0/R_0^2, the Sun gets the largest anomalous acceleration in the solar system. These accelerations will modify the orbits of the planets. In particular, they will cause misalignments of the orbital planes if the anomalous accelerations are out of the ecliptic plane. In comparison, the typical relative accelerations between the planets and the Sun are $10^{-34} - 10^{-38}$ GeV, the Viking ranging data constrains any anomalous radial acceleration acting on Earth or Mars to be smaller than $\sim 10^{-44}$ GeV while the Pioneer anomaly corresponds to $a_{\rm Pioneer} \sim 10^{-42}$ GeV [31]. If we require the anomalous accelerations to be smaller than $\sim 10^{-44}$ GeV, we obtain an upper bound on M,

$$M \lesssim 10^{10}(\alpha\kappa)^{-1/2}\ \text{GeV}. \tag{6.11}$$

Another potential constraint outside the solar system comes from the period change of the binary pulsar system. note that even if the velocities of the pulsars are smaller than c_s, the Goldstone boson can still be emitted through the quadrupole radiation. the energy loss can be calculated in a uniform way. To simplify the problem, we assume that two pulsars of equal mass M_0 moving in a circular orbit with distance $2r_0$ apart. We also ignore the velocity of the center of the system.

In the δ-function approximation, the aource is given by

$$\rho(x,t) = M_0\delta^3\left(\vec{x}-\vec{r}(t)\right) + M_0\delta^3\left(\vec{x}+\vec{r}(t)\right), \tag{6.12}$$

$$\vec{r}(t) = (r_0\cos\omega_0 t,\ r_0\sin\omega_0 t,\ 0), \tag{6.13}$$

where the angualr frequency ω_0 is related to the orbital velocity v_0 by $v_0 = \omega_0 r_0$. Its Fourier transform is

$$\tilde{\rho}(\omega,k) = 2M_0\sum_{n=-\infty}^{\infty} e^{i\,2n(\theta_k-\frac{\pi}{2})}J_{2n}(k_\parallel r_0)\,2\pi\delta(\omega-2n\omega_0), \tag{6.14}$$

where $k_\parallel = \sqrt{k_x^2+k_y^2}$, $\tan\theta_k = k_y/k_x$ and we have used the property of the Bessel functions, $J_m(-x) = (-1)^m J_m(x)$. A form factor $f(k)$ can also be included to represent the finite size effect. Following the same procedure we obtain the time-averaged energy loss rate

$$\frac{dE_S}{dt} = -\frac{\alpha M_0^2 M^2}{4\pi M_{\rm Pl}^4 c_s}\sum_{n=-\infty}^{\infty}\int_0^1 d\cos\theta\left(\frac{2n\omega_0}{c_s}\right)^2 J_{2n}\left(\frac{2n\omega_0 r_0\sin\theta}{c_s}\right)^2\left|f\left(\frac{2n\omega_0}{c_s}\right)^2\right|. \tag{6.15}$$

Now we can consider two different limits. First, for $v_0 = \omega_0 r_0 \ll c_s$ we can use the approximation

$$J_m(x) \approx \frac{1}{m!}\left(\frac{x}{2}\right)^m, \quad \text{for small } x. \tag{6.16}$$

The sum is dominated by the smallest $|n|$, $n = \pm 1$ which corresponds to the quadrupole radiation. There is no form factor suppression ($f(k) \approx 1$) because the inverse of the momentum is much bigger than the size of the system. the resulting energy loss by the quadrupole radiation is

$$\frac{dE_S}{dt} = -\frac{4\alpha M_0^2 M^2 v_0^6}{15\pi M_{\rm Pl}^4 r_0^2 c_s^7} = -\frac{2^{12}\pi\alpha M^2 v_0^{10}}{15\,c_s^7}, \tag{6.17}$$

where in the last equality we have used the relation $G_N M_0/r_0 = 4v_0^2$. On the other hand, the energy loss due to the usual gravitational wave is given by

$$\frac{dE_{\rm tensor}}{dt} = -\frac{1}{8\pi M_{\rm Pl}^2}\frac{1}{5}\langle\dddot{Q}\dddot{Q}\rangle = -\frac{2}{5}\frac{G_N^4 M_0^5}{r_0^5} = -\frac{2^{14}\pi M_{\rm Pl}^2 v_0^{10}}{5}. \tag{6.18}$$

Taking the ratio, we have

$$\frac{dE_S/dt}{dE_{\rm tensor}/dt} = \frac{\alpha M^2}{12 M_{\rm Pl}^2 c_s^7} = \frac{\alpha_2^{\rm PPN}}{6\, c_s^3} \approx 10^2 \left(\frac{\alpha_2^{\rm PPN}}{4\times 10^{-7}}\right)\left(\frac{10^{-3}}{v_0}\right)^3 \left(\frac{v_0}{c_s}\right)^3. \tag{6.19}$$

The observed period change of the binary pulsars PSR 1913+16 is consistent with the energy loss entirely due to the gravitational wave radiation, so the above ratio must be smaller than 1. Given their orbital velocity $v_0 \sim 10^{-3}$, we can see that for $v_0 \ll c_s$ (say, $c_s > 10\, v_0$), this does not give a stronger bound than the $\alpha_2^{\rm PPN}$ bound from the solar system. For v_0 getting close to c_s, this can compete with the solar system bound, but our approximation starts to break down.

Next, let us consider the opposite limit, $c_s \ll v_0$. Now we need to use the asymptotic approximation of the Bessel functions,

$$J_m(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left[x - \left(m + \frac{1}{2}\right)\frac{\pi}{2}\right], \quad \text{for } x \gg 1. \tag{6.20}$$

From Eq. (6.15) the energy loss is given approximately by

$$\frac{dE_S}{dt} \approx -\frac{\alpha M^2 M_0^2}{4\pi M_{\rm Pl}^4 c_s r_0^2} \sum_{n=1}^{\infty} \frac{2nv_0}{c_s} \left| f\left(\frac{2nv_0}{c_s r_0}\right)^2 \right|. \tag{6.21}$$

The problem is that the arguments in the form factor take values much bigger than $1/r_0$ in this case. As we discussed before. nonlinear effects are already important at that length scale and the region of the ether being dragged by the pulsars is approximately of size r_0. The effective form factor must be very suppressed and is difficult to calculate from first principles. Physically, The binary system emits ether waves with angular frequencies that are integer mutiples of $2\omega_0$. Because the sound velocity of the ether is much smaller than the orbital velocity, the wavelengths of the ether waves are much shorter than the size of the system. Therefore the amplitudes of the ether waves must be highly suppressed. If we assume that $f(2nv_0/c_s r_0) \sim (2nv_0/c_s)^{-p}$ with $p > 1$, then the sum in Eq. (6.21) is $\sim (c_s/v_0)^{2p-1}$, and the energy loss would be

$$\frac{dE_S}{dt} \sim -\frac{\alpha M^2 M_0^2}{4\pi M_{\rm Pl}^4 r_0^2 c_s}\left(\frac{c_s}{v_0}\right)^{2p-1} \sim 10^3 \alpha \left(\frac{M}{10^{10}\ \text{GeV}}\right)\left(\frac{10^{-3}}{v_0}\right)^7 \left(\frac{c_s}{v_0}\right)^{2p-2} \frac{dE_{\rm tensor}}{dt}. \tag{6.22}$$

Obviously for $c_s \ll v_0$ this does not give a significant constraint. For c_s getting close to v_0 it may become a competitive constraint depending on the power of the suppression factor, but again our approximation starts break down.

7 Couplings to the Standard Model

Given that gauged ghost condensation yields such a modest modification of gravity, the strongest bounds on this model of spontaneous Lorentz violation could come from direct couplings between $\mathcal{A}_\mu$ and the Standard Model. As we will see, most couplings can be forbidden by discrete symmetries, such that from an experimental point of view, there would be virtually no way of distinguishing between a Lorentz invariant vacuum and a vacuum with non-zero $\langle \mathcal{A}_\mu \rangle$. While this is a theoretically plausible possibility, it not a particularly interesting one, so we want to consider what would happen if the gauged ghost condensate were not confined to a hidden sector.

Even in the absence of gravitational couplings, the Lagrangian in Eq. (3.21) is quite fascinating because it is the effective field theory description of an Abelian gauge theory in a Lorentz-violating Higgs phase at distances long compared to the Higgs scale. To our knowledge, the dynamics of this phase has not been studied in the literature. Like the Lorentz invariant Higgs phase, the gauge boson eats a Goldstone and therefore has three polarizations, but what is especially bizarre about this system is that all three polarizations are massless. There are long-range "magnetic" interactions in this theory but "electric" interactions are exponentially suppressed. In other words, in order to produce a healthy Newton's law for gravity, we had to sacrifice Coulomb's law for this ghost-electromagnetic $U(1)$. **(Comments about papers by Kostelecky,...on EM from spontaneous Lorentz violation)**

7.1 Catalog of Allowed Couplings

Even if we do not include direct interactions between the Satndard Model fields and $\mathcal{A}_\mu$ in the first place, we still expect graviton loops to generate interactions of the form

$$\Delta\mathcal{L} \sim c\frac{M^2}{M_{\rm Pl}^4} T^{\mu\nu}\mathcal{A}_\mu\mathcal{A}_\nu, \tag{7.1}$$

where $T_{\mu\nu}$ is a symmetric dimension four Standard Model operator, and c is expected to be an $\mathcal{O}(1)$ coefficient. The $1/M_{\rm Pl}^4$ suppression arises because $\mathcal{A}_\mu$ contains $\partial_\mu\phi$ and therefore this interaction is actually dimension eight, and the factors of M are inserted because we have defined $\mathcal{A}_\mu$ to have mass dimension $+1$. If we set $\mathcal{A}_\mu$ to its vacuum expectation value $\langle \mathcal{A}_0 \rangle = M$, then our Lagrangian contains the operator

$$\Delta\mathcal{L} \sim c\frac{M^4}{M_{\rm Pl}^4} T^{00}. \tag{7.2}$$

Taking $T^{\mu\nu}$ to be the stress-energy tensor for a Standard Model fermion, then the inclusion of T^{00} allows the maximal attainable velocity of the fermion to differ from the speed of light:

$$\delta v_\psi \sim \frac{M^4}{M_{\rm Pl}^4}. \tag{7.3}$$

Constraints from high precision spectroscopy and the absence of vacuum Cerenkov radiation bound $\delta v_\psi \lesssim 10^{-21} - 10^{-23}$ [32]. This limits the scale of spontaneous Lorentz violation to be $M \lesssim 10^{13}$ GeV. Remarkably, this bound on M is *stronger* than the bound from the gravitational sector, in contrast with (ungaged) ghost condensation.

Now we consider the direct couplings of the Standard model fields to the Lorentz-violating sector linear in $\mathcal{A}_\mu$. Since $\mathcal{A}_\mu = A_\mu + M\partial_\mu\phi$ is a $U(1)_{\rm ghost}$ gauge invariant combination, it can couple to currents made of Standard Model fields. In fact, this is just equivalent to that the Standard Model fields are charged under this $U(1)_{\rm ghost}$ gauge symmetry. Consider the interaction

$$\tilde{q}_\psi\, \mathcal{A}_\mu \bar{\psi}\gamma^\mu\psi = \tilde{q}_\psi\,(A_\mu + M\partial_\mu\phi)\bar{\psi}\gamma^\mu\psi, \tag{7.4}$$

where ψ is a Standard Model fermion. The interaction with $\partial_\mu\phi$ can be removed by a field redefinition

$$\psi \to \psi' = e^{-iqM\phi}\psi \tag{7.5}$$

into the kinetic term. In terms of the redefined field ψ', the interaction Eq. (7.4) becomes

$$\tilde{q}_\psi A_\mu \bar{\psi}'\gamma^\mu\psi' \tag{7.6}$$

which is just the $U(1)_{\rm ghost}$ gauge interaction of a fermion with charge $\tilde{q}_\psi$. The point is that $\exp(-iM\phi)$ carries a unit charge of the $U(1)_{\rm ghost}$ gauge symmetry, so the charge of a field under this $U(1)_{\rm ghost}$ can be altered by multiplying a certain power of $\exp(-iM\phi)$. It is most convenient, however, to work in the ψ' basis where $\partial_\mu\phi$ coupling is removed.

A new long-range force other than electromagnetism and gravity is strongly constrained. In gauged ghost condensation the $\mathcal{A}_0$ fieid is massive, so there is no $1/r$ potential between $U(1)_{\rm ghost}$ charges in the preferred rest frame. Nonetheless, $\mathcal{A}_i$ is still massless, giving rise to additional magnetic forces. Effectively, this gives a tree-level contribution to the magnetic moment of a particle charged under the $U(1)_{\rm ghost}$. If we assume that the $U(1)_{\rm ghost}$ charges of the Standard Model fields are proportional to their $U(1)_{\rm EM}$ charges, the correction to the Landé g factor is

$$\delta\left(\frac{g-2}{2}\right) \sim \frac{\tilde{q}_e^2\alpha_{\rm ghost}}{\alpha_{\rm EM}}, \tag{7.7}$$

where $\alpha_{\rm ghost} = g^2/(4\pi)$. Note that in theories with a normal para-$U(1)$ gauge group that "regauges" electromagnetism [40], this correction is absent, because the para-$U(1)$ also shifts the electrostatic charges of the fermions. The most accurately measured $g-2$ is for the electron with an uncertainty at the 10^{-9} level. However, since it serves as a definition of $\alpha_{\rm EM}$, to constrain $\tilde{q}_e g$ we need the next most precise determination of $\alpha_{\rm EM}$. A measurement of $\alpha_{\rm EM}$ using the atom interferometry with the laser-cooled cesium atoms has reached the 10^{-8} uncertainty level and the result is in agreement with the $g-2$ measurement [?]. This implies a constraint

$$\frac{\tilde{q}_e^2\alpha_{\rm ghost}}{\alpha_{\rm EM}} \lesssim 10^{-8} \qquad \Longrightarrow \qquad \tilde{q}_e g \lesssim 10^{-4}. \tag{7.8}$$

This bound is much weaker than the bound from the long-range force derived in the next subsection: for sources moving with respect to the preferred frame, there is a direction-dependent pseudo-Coulomb potential. If the $U(1)_{\rm ghost}$ charges are not proportional to the electromagnetic charges, then the measurement of $(g-2)$ would depend on which species of fermion was responsible for setting up the background magnetic field, though in practice, almost all laboratory magnetic fields are in some way established by electrons.

One can also consider the couplings of the Standard Model antisymmetric tensors to the field tensor of the $U(1)_{\rm ghost}$. At the lowest order, we can write down a kinetic mixing between the $U(1)_{\rm ghost}$ and $U(1)_{\rm EM}$,

$$\Delta\mathcal{L} = c_o F_{\mu\nu} F^{\mu\nu}_{\rm EM}. \tag{7.9}$$

Such a term can be removed by a redefinition of the $U(1)$ gauge fields, resulting in a shift of the $U(1)_{\rm ghost}$ charges of the Standard Model fields. So, the effect is the same as discussed in the previous paragraph. Alternatively, we could imagine dipole interactions

$$\Delta\mathcal{L} = \frac{1}{\Lambda}\sum_{\Psi} c_\Psi \bar{\Psi}\sigma^{\mu\nu}\Psi F_{\mu\nu}, \tag{7.10}$$

These couplings were recently studied in a Lorentz invariant setting in Ref. [41], where the bound on Λ for the electron was expressed in terms of the Higgs vev $\langle h\rangle = 174$ GeV:

$$\frac{\Lambda}{c_e} > \frac{(3-60\ \text{TeV})^2}{\langle h\rangle} \sim 10^5 - 10^7\ \text{GeV}, \tag{7.11}$$

where the more stringent bound comes if Eq. (7.10) introduces flavor mixing between electrons and muons. It would be interesting to study these couplings more carefully in the Lorentz-violating Higgs phase, but we do not expect significant departures from these bounds.

To forbid the coupling in Eq. (7.4), we could impose an $\mathcal{A}_\mu \to -\mathcal{A}_\mu$ symmetry on the theory, but this would restrict all couplings between $\mathcal{A}_\mu$ and the Standard Model to be of the form of Eq. (7.1). One way to forbid a coupling to the electomagnetic current is to choose ϕ to be parity-odd, or equivalently

$$P: \qquad \mathcal{A}_0 \to -\mathcal{A}_0, \qquad \mathcal{A}_i \to \mathcal{A}_i. \tag{7.12}$$

In the Standard Model, electroweak interactions violate parity, and because we expect the scale M to be much higher than the electroweak scale, at best we can say that the coupling of $\mathcal{A}_\mu$ to parity-even currents should be suppressed relative to partity-odd currents. For a fermion Ψ with Dirac mass m_D, we can construct the vector and axial currents:

$$J_\mu = \bar{\Psi}\gamma_\mu\Psi, \qquad J^5_\mu = \bar{\Psi}\gamma_\mu\gamma^5\Psi. \tag{7.13}$$

By looking at the relevant triangle diagram, we estimate that the coupling $\mathcal{A}^\mu J_\mu$ should be suppressed relative to the coupling $\mathcal{A}^\mu J^5_\mu$ by $m_D^2 G_F/16\pi^2$, where G_F is Fermi's constant. For an electron, this suppression is 10^{-14}, so to a good approximation we can ignore couplings to parity-even currents.

With the above caveats, the leading coupling of $\mathcal{A}_\mu$ to any light Dirac fermion, in particular electrons and nucleons, is

$$\mathcal{L}_{\rm int} = \frac{M}{F}\mathcal{A}^\mu J^5_\mu, \tag{7.14}$$

where we parametrize the coefficient as a ration of the two scales M and F to compare with the result of ungauged ghost condensation. Setting $\mathcal{A}_\mu$ to its vev:

$$\mathcal{L}_{\rm int} = \mu\bar{\Psi}\gamma^0\gamma^5\Psi, \qquad \mu = \frac{M^2}{F}. \tag{7.15}$$

This Lorentz- and CPT-violating term is the same as in (ungauged) ghost condensation [4, 5], and gives rise to different dispersion relations for the left- and right-handed fermions [33, 34]. In a frame boosted with respect to the preferred frame, Eq. (7.15) contains the interaction

$$\mu\vec{s}\cdot\vec{v}_{\rm earth}, \tag{7.16}$$

where we have identitified $\bar{\Psi}\vec{\gamma}\gamma^5\Psi$ with non-relativistic spin density $\vec{s}$. Assuming the preferred rest frame is aligned with the CMBR, we take $|\vec{v}_{\rm earth}| \sim 10^{-3}$, and experimental bounds on velocity-dependent spin couplings to electrons is $\mu \sim 10^{-25}$ GeV [35] and to nucleons $\mu \sim 10^{-24}$ GeV [36, 37].

After integrating out $\mathcal{A}_0$, we are left with the interaction

$$\mathcal{L}_{\rm int} = \frac{M}{F}\vec{s}\cdot\vec{\mathcal{A}}. \tag{7.17}$$

The $g \to 0$ limit of this coupling was discussed in Ref. [5] and it led to two Lorentz-violating dynamical effects: "ether" Cerenkov radiation and a long-range spin-dependent potential. These effects in this model will be discussed in the next subsection.

7.2 Dynamics of the Lorentz-Violating Higgs Phase

The Lagrangian in Eq. (3.15) is the most general Lagrangian with an $\mathcal{A}_\mu \to -\mathcal{A}_\mu$ symmetry that is spontaneously broken by the vacuum $\langle\mathcal{A}_0\rangle = M$. If we only enforce Eq. (7.12), then we can add additional couplings to the gauged ghost Lagrangian such as

$$\Delta\mathcal{L} \sim M(\mathcal{A}_0 - M)\partial_i\mathcal{A}^i. \tag{7.18}$$

When we integrate out $\mathcal{A}_0$, however, this term does not change the basic form of Eq. (3.21), and the effective Lagrangian for $\mathcal{A}_i$ at distances large compared to $1/M$ is

$$\mathcal{L}_{\rm eff} = \frac{1}{2c_v^2 g^2}(\partial_t\vec{\mathcal{A}})^2 - \frac{1}{4g^2}F_{ij}^2 - \tfrac{1}{2}\alpha(\vec{\nabla}\cdot\vec{\mathcal{A}})^2, \tag{7.19}$$

where we have absorbed the effect of β_1 and α_2 into the parameters g and c_v, and c_v has the interpretation of the velocity of the transverse vector modes.

Going to $\mathcal{A}_i \equiv A_i$ gauge, it is straightforward to calculate the A_iA_j propagator,

$$\langle A_i A_j \rangle = \frac{1}{(\omega/c_v)^2 - k^2}\left(g^2\delta_{ij} + (g^2 - 1/\alpha)\frac{k_i k_j}{(\omega/c_s)^2 - k^2}\right), \tag{7.20}$$

where $c_s = c_v g\sqrt{\alpha}$ is the velocity of the longitudinal mode of A_i. We will always assume that $c_s \ll c_v$. The first thing to check is what kind of pseudo-Coulomb potential is generated by the interaction in Eq. (**??**) in a frame moving with respect to the preferred frame. Consider a source of charge $\tilde{Q}$ under $U(1)_{\text{ghost}}$ moving with velocity $v < c_v$, c_s:

$$\begin{aligned} J^0_{EM} &= \tilde{Q}\delta^{(3)}(\vec{x} - \vec{v}t) & \tilde{J}^0_{EM} &= 2\pi\tilde{Q}\delta(\omega - \vec{k}\cdot\vec{v}) \\ \vec{J}_{EM} &= \tilde{Q}\vec{v}\delta^{(3)}(\vec{x} - \vec{v}t) & \tilde{\vec{J}}_{EM} &= 2\pi\tilde{Q}\vec{v}\delta(\omega - \vec{k}\cdot\vec{v}) \end{aligned} \tag{7.21}$$

where there are corrections at $\mathcal{O}(v^2)$. To this order, we can use the $\omega \to 0$ limit of Eq. (7.20). Taking the Fourier transform of the propagator, the potential between $\tilde{Q}$ and a test charge $\tilde{q}$ a comoving distance r away is

$$V(r,\theta) = -\frac{g^2 v^2 \tilde{Q}\tilde{q}}{8\pi r}\left(1 + \cos^2\theta + \frac{1}{g^2\alpha}\sin^2\theta\right), \tag{7.22}$$

where $\vec{r}\cdot\vec{v} = \cos\theta$. In the limit $c_s \ll c_v$, the third term dominates, and we get an effective anomalous coupling

$$g_{\text{anom}} \sim \frac{v}{\sqrt{\alpha}}, \tag{7.23}$$

and a fascinating angular dependence with respect to the velocity of the preferred frame. Bounds on new gauge interactions are usually derived by noting the absence of any long-range forces other than electromagnetism and gravity [ref?]. In particular, precision tests of gravity using materials with different baryon-to-lepton ratios place limits on a possible $U(1)_{B-L}$ gauge coupling of $g_{B-L} \ll 10^{-19}$ [ref?]. If we assume that searches for $U(1)_{B-L}$ gauge couplings would be sensitive to such velocity effects, then as quoted before, $g_{\text{anom}} < 10^{-19}$. NEED A BETTER REFERENCE WITH ANGULAR DEPENDENCE. If we assume that the earth is moving with velocity $|\vec{v}| \sim 10^{-3}$ with respect to the preferred rest frame, then

$$\tilde{q}_\psi \lesssim 10^{-16}\sqrt{\alpha}, \tag{7.24}$$

where ψ represents ordinary particles p, n, e.

The coupling in Eq. (7.14) between $\vec{A}$ and spin is familiar from (ungauged) ghost condensation. Consider spins $\vec{S}_1$ and $\vec{S}_2$ separated by a distance r that are at rest with respect to the preferred frame. The $1/r$ potential between them takes the form

$$V(r) = -\frac{M^2}{F^2}\frac{1}{8\pi r}\left((g^2 + 1/\alpha)\vec{S}_1\cdot\vec{S}_2 + (g^2 - 1/\alpha)(\vec{S}_1\cdot\hat{r})(\vec{S}_2\cdot\hat{r})\right). \tag{7.25}$$

Note that this reduces to the pure Goldstone result in the $g \to 0$ limit, modulo a redefinition $M \to \sqrt{\alpha} M$. This potential has a very interesting feature which distinguishes it from both electromagnetism and (ungauged) ghost condensation. Consider a toroidal solenoid filled with a ferromagnetic material.[5] When current runs through the solenoid, the ferromagnetic spins will align in the azimuthal direction, but as we will show in Appendix E, there will be no net magnetic field nor no net longitudinal $\vec{A}$ field outside of the solenoid. However, there will be transverse $\vec{A}$ fields outside the solenoid which can interact with a test spin. Because there is no magnetic field leakage from this geometry, this allows for a null test for the inverse-square law spin-dependent force.

In order for the potential in Eq. (7.25) to be experimentally testable, it has to be at least roughly of gravitational strength, and if we assume spin sources with one aligned spin per nucleon, M/F has to be of order $m_N/M_{\rm Pl} \sim 10^{-19}$. As shown in Ref. [5], M/F cannot be much larger than 10^{-19} without the theory exiting its range of validity. When M/F is so small, it is easy to satisfy the condition in Eq. (7.24). In the previous section, we argued that the ratio of the vector coupling to the axial coupling can be made as small as $m_e^2 M_{\rm Pl}/16\pi^2 \sim 10^{-14}$ without fine tuning if we assume the quasi-parity invariance in Eq. (7.12). This tells us that

$$\frac{\tilde{q}_\psi}{M/F} < 10^{-14} \qquad \Longrightarrow \qquad \tilde{q}_\psi < 10^{-35} \tag{7.26}$$

in agreement with Eq. (7.24).

This spin-spin potential is even more interesting at finite velocity. In the case $v < c_s$, we can calculate the spin-spin potential as a power series in v/c_s. For simplicity, we define

$$f(\vec{A}, \vec{B}) = \vec{A} \cdot \vec{B} - (\vec{A} \cdot \hat{r})(\vec{B} \cdot \hat{r}). \tag{7.27}$$

The potential then takes the form

$$\begin{aligned} V(r) = &-\frac{g^2}{4\pi r} \vec{S}_1 \cdot \vec{S}_2 - \frac{1/\alpha - g^2}{8\pi r} f(\vec{S}_1, \vec{S}_2) \\ &- \frac{1/\alpha - g^2}{32\pi r} \frac{v^2}{c_s^2} \Big(f(\vec{S}_1, \vec{S}_2) f(\hat{v}, \hat{v}) + 2 f(\vec{S}_1, \hat{v}) f(\vec{S}_2, \hat{v}) \Big) + \mathcal{O}(v^4/c_s^4). \end{aligned} \tag{7.28}$$

This potential contains an incredible amount of angular information. WHAT MORE TO SAY?

Next we consider the energy loss due to the emission of $\mathcal{A}_i$ particles. In Ref. [5] we did not consider bounds on new light particles from astrophysics [38], because the scale of spontaneous Lorentz violation M was constrained to be much lower than typical astrophysical energies. Now that we can raise the scale of Lorentz violation well above even the electroweak

[5] Thanks to Blayne Heckel for suggesting this geometry.

scale, these bounds are relevant, but its easy to see that once we impose the condition on μ, the astrophysical bound is almost automatically satisfied. For a pseudo-scalar φ coupling to the axial current, stellar energy loss arguments bound the coefficient of the operator

$$\frac{1}{F} J^5_\mu \partial^\mu \varphi \tag{7.29}$$

to be $F > 10^9$ GeV [39]. In the non-relativistic limit, Eq. (7.29) is identical to the canonically normalized coupling of the longitudinal mode of $\vec{\mathcal{A}}$ to spin-density, so as long as $M \gtrsim 10$ eV, the bound on μ is stricter than the bound on F directly. For the transverse modes of $\vec{\mathcal{A}}$, we can estimate the bound on M/F by going to canonical normalization and replacing the derivative ∂^μ with a typical astrophysical energy $E_{\text{astro}} \sim 100$ keV:

$$\frac{1}{F} J^5_\mu \partial^\mu \varphi \sim \frac{E_{\text{astro}}}{F} \vec{J}^5 \cdot \vec{A}^c \qquad \Longrightarrow \qquad \frac{M}{F} g < 10^{-13}. \tag{7.30}$$

As long as $M \gtrsim 10^{-3}$ eV$/g$, the bound on μ enforces the astrophysical bound. This bound also applies to the vector coupling, requiring

$$\tilde{q}_\psi g < 10^{-13}. \tag{7.31}$$

Because the scalar mode in gauged ghost condensation has a finite velocity, this Cerenkov effect is turned off when the velocity of the source v is smaller than c_s. *Check the following arguments.* When $v/c_s \gg 1$, we expect the discussion of Ref. [5] to go through virtually unchanged, where we found that within the range of validity of the effective theory, there were no observable effect from ether Cerenkov radation.

8 Conclusion

We have presented a low energy effective theory of spontaneous Lorentz violation where the scale of breaking M can be much higher than particle physics energies. Looking at just the gravity sector, we found that $M \lesssim 10^{15}$ GeV in order to satisfy experimental constraints on the PPN parameters, and if we assume the most naïve allowed couplings to the Standard Model, graviton loop effects suggest $M \lesssim 10^{13}$ GeV. Amazingly, by taking the $g \to 0$ of this incredibly healthy theory, we recover the Lagrangian of (ungauged) ghost condensation, in which the scale of spontaneous Lorentz violation had to be less than 10 MeV due to a gravitational instability. Indeed, without the example of ghost condensation, it would seem very unlikely that we could ever push the gauge coupling g below $g_c \sim M/M_{\text{Pl}}$. However, by understanding the power counting, we see that the $g \ll g_c$ and $g \gg g_c$ limits of this theory are both well-defined.

This construction now allows two equally consistent ways to complete a non-relativistic theory into a relativistic one. The method familiar from relativistic field theory is to simply replace spatial derivatives with space-time derivatives. However, certain sectors may not want to be Lorentz invariant at low energies, in these cases one can simply introduce the Goldstone boson of spontaneous Lorentz violation. If the Goldstone is the only new degree of freedom, then it mixes dangerously with the Newtonian potential, but we have seen that if we allow the Goldstone to shift under a new $U(1)$ gauge transformation, this mixing effect can be removed for large enough gauge coupling. An example where this is relevant is the proposal of Ref. [42] for relativistic modified Newtonian dynamics (MOND). MOND theories appear to work best when there is a preferred cosmic frame, and while it is possible to use a Lagrange multiplier constraint (à la Ref. [24]) to define a vector field with a preferred time-like orientation, it is not clear how to do power counting with Lagrange multipliers. Gauged ghost condensation defines a preferred reference frame in a framework where the distinction between relevant and irrelevant operators is well-defined.

WHAT MORE?

A Decoupling Limit and Comparison with the Literature

Alternative theories of gravity with additional vector fields pointing in some preferred direction have been considered in the literature [**?**]. The value of the vector field is often imposed as a constraint through a Lagrange multipler

$$\mathcal{L} \supset -\kappa(u^\mu u_\mu - 1). \tag{A.1}$$

We will see that it corresponds to certain decoupling limit of our theory. To see this, we can add a term $-2\kappa^2/M^4$ to the Lagrange multiplier. Integrating out κ we obtain

$$\mathcal{L} \supset \frac{M^2}{8}(u^2 - 1)^2 \tag{A.2}$$

which is exactly the leading term of our gauged ghost condensation Lagrangian. The constraint of the Lagrange multiplier is obtained by sending $M \to \infty$. Therefore, we see that these theories correspond to the decoupling limit $M \to \infty$ of gauged ghost condensation, with appropriate rescaling of other parameters, $g^{-2}, \alpha, \beta, \gamma, \ldots \to 0$ while keeping $g^{-2}M^2, \alpha M^2, \beta M^2, \gamma M^2, \ldots$ finite.

In this subsection we would like to study this decoupling limit. We start with the ungauged ghost condensate with the lagrangain

$$\mathcal{L} = \mathcal{L}_{\rm EH} + \tfrac{1}{8}M^4(X-1)^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2, \tag{A.3}$$

where $\mathcal{L}_{\rm EH}$ is the Einstein-Hilbert Lagrangian. For convenience we have defined $\tilde{M}^2 = \alpha M^2$. We want to understand the physics of taking $M \to \infty$, with $\tilde{M}$ and $M_{\rm Pl}$ held fixed. This is equivalent to imposing the constraint $X = 1$.

The linearized Lagrangian for the scalar sector including a matter source ρ is

$$\mathcal{L} = -M_{\rm Pl}^2(\vec{\nabla}\Phi)^2 + \tfrac{1}{2}M^4(\Phi - \dot{\pi})^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2 + \rho\Phi. \tag{A.4}$$

For large M, we impose the constraint $\Phi = \dot{\pi}$, which gives the effective Lagrangian

$$\mathcal{L}_{\rm eff} = -M_{\rm Pl}^2(\vec{\nabla}\dot{\pi})^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2 + \rho\dot{\pi}. \tag{A.5}$$

From this we easily read off the dispersion relation for the scalar mode:

$$\omega^2 = -\frac{\tilde{M}^2}{2M_{\rm Pl}^2}\vec{k}^2. \tag{A.6}$$

Note that this has the wrong sign for $\tilde{M}^2 > 0$, which is just a manifestation of the Jeans instability in ghost condensation. But for $\tilde{M}^2 < 0$, the scalar has a healthy dispersion relation.

Of course, the same results can be obtained from the dispersion relation for π with finite M from Eq. (4.1). Rewriting in terms of $\tilde{M}$:

$$\omega^2 = -\frac{\tilde{M}^2}{2M_{\rm Pl}^2}\vec{k}^2 + \frac{\tilde{M}^2}{M^4}\vec{k}^4. \tag{A.7}$$

In the limit $M \to \infty$ only the first term survives. We see that this limit is equivalent to

$$|\vec{k}| \ll m = \frac{M^2}{\sqrt{2}M_{\rm Pl}}. \tag{A.8}$$

We therefore see that we can define a healthy theory as long as m is large enough to be a UV scale and if we change the sign of $\tilde{M}^2$ relative to the ghost condensate. However, the theory does not have a Newtonian potential in this limit! The $\omega \to 0$ limit of the $\Phi\Phi$ propagator is

$$\langle\Phi\Phi\rangle \to -\frac{1}{2M_{\rm Pl}^2}\frac{1}{\vec{k}^2 - M^4/2M_{\rm Pl}^2} \simeq \frac{1}{M^4}, \tag{A.9}$$

which is local in space. So this is not a viable modification of gravity.

Now consider the gauged ghost condensate. To make the limit clearer, we define π to have mass dimension -1, A_μ to have mass dimension $+1$, and rescale the fields and couplings relative to Eq. (4.3) by

$$A_\mu \to \frac{M}{\mu}A_\mu, \quad \frac{1}{g^2} \to \left(\frac{\mu}{M}\right)^2 \frac{1}{g^2}, \tag{A.10}$$

where μ is a mass scale held fixed when we take the limit $M \to \infty$. The gauge transformations can now be writtten as

$$\delta A_\mu = \partial_\mu \lambda, \qquad \delta\pi = -\frac{\lambda}{\mu}, \tag{A.11}$$

and the scalar sector Lagrangian is

$$\mathcal{L} = -M_{\rm Pl}^2(\vec{\nabla}\Phi)^2 + \frac{1}{2g^2}(\vec{\nabla}A_0)^2 + \tfrac{1}{2}M^4(\Phi - \dot{\pi} + A_0/\mu)^2 - \tfrac{1}{2}\tilde{M}^2(\vec{\nabla}^2\pi)^2. \qquad \text{(A.12)}$$

We are interested in the limit $M \to \infty$ with the other couplings (including μ) held fixed. The dispersion relation for the scalar mode is

$$\omega^2 = -\frac{\tilde{M}^2}{2M_{\rm Pl}^2}\left(1 - \frac{2g^2 M_{\rm Pl}^2}{\mu^2}\right)\vec{k}^2 + \frac{\tilde{M}^2}{M^4}\vec{k}^4. \qquad \text{(A.13)}$$

As before, the second term is absent in the limit $M \to \infty$, and is neglible for $|\vec{k}| \ll m$. The $\vec{k}^2$ term has the right sign for

$$\tilde{M}^2 > 0, \qquad g > g_{\rm c} = \frac{\mu}{\sqrt{2}M_{\rm Pl}}, \qquad \text{(A.14)}$$

or for $\tilde{M}^2 < 0$, $g < g_{\rm c}$. It is easy to see that in order to get a viable Newtonian potential, we must choose the first option. The static limit of the $\Phi\Phi$ propagator is

$$\langle\Phi\Phi\rangle = -\frac{1}{2M_{\rm Pl}^2}\frac{1}{\vec{k}^2}\frac{1}{1 - g_{\rm c}^2/g^2}, \qquad \text{(A.15)}$$

so if $g < g_{\rm c}$ this would have the wrong sign relative to the standard Newtonian potential.

WHAT ARE WE DOING WITH THIS SECTION? Therefore to get a viable $M \to \infty$, we have to do GGC not Jacobson?

B Stress-energy tensor of the gauged ghost condensate

The goal of this appendix is to calculate the stress-energy tensor of the gauged ghost condensate. For this purpose we need to consider general variations of the metric since the stress-energy tensor is defined as the functional derivative of an action with respect to the metric components. On the other hand, in order to make the expression independent of the heavy modes we have to integrate out the perturbation of $D_0\phi$ before taking the variation and, thus, we need to separate the time coordinate and space coordinates. For these reasons, we shall adopt the ADM decomposition of the metric:

$$ds^2 = -N^2 dt^2 + q_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \qquad \text{(B.1)}$$

and define $\mathcal{A}_\mu$ by

$$D_0\phi = (1 + \mathcal{A}_0)N, \quad D_i\phi = \mathcal{A}_i. \qquad \text{(B.2)}$$

We shall integrate $\mathcal{A}_0$ out.

Expanding $X-1$ as

$$\frac{1}{2}(X-1) = \mathcal{A}_0 - \frac{1}{2}\left(q^{ij} - \frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_i\mathcal{A}_j + O(\mathcal{A}_0^2), \tag{B.3}$$

the leading Lagrangian is

$$\mathcal{L} = \frac{M^4}{2}\left[\mathcal{A}_0 - \frac{1}{2}\left(q^{ij} - \frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_i\mathcal{A}_j\right]^2 - \frac{M^2}{4g^2}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha M^2}{2}(\nabla^\mu D_\mu\phi)^2, \tag{B.4}$$

where $q^{ij} = (q^{-1})^{ij}$ and $q = \det q$. Integrating out the massive mode $\mathcal{A}_0$,

$$\mathcal{A}_0 = \frac{1}{2}\left(q^{ij} - \frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_i\mathcal{A}_j, \tag{B.5}$$

we obtain

$$F_{\mu\nu}F^{\mu\nu} = -2q^{ij}F_{\perp i}F_{\perp j} + q^{ik}q^{jl}F_{ij}F_{kl}, \tag{B.6}$$

and

$$\nabla^\mu D_\mu\phi = -\frac{1}{\sqrt{q}}\partial_\perp\left\{\sqrt{q}\left[1 - \frac{\beta^j\mathcal{A}_j}{N} + \frac{1}{2}\left(q^{kl} - \frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\right\} + \frac{1}{N\sqrt{q}}\partial_i(N\sqrt{q}q^{ij}\mathcal{A}_j), \tag{B.7}$$

where

$$\partial_\perp \equiv \frac{1}{N}(\partial_t - \beta^i\partial_i), \tag{B.8}$$

and

$$\begin{aligned} F_{\perp i} &\equiv \frac{1}{N}(F_{ti} - \beta^k F_{ki}) \\ &= \partial_\perp\mathcal{A}_i - \frac{1}{N}\partial_i\left\{N\left[1 + \frac{1}{2}\left(q^{kl} - \frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\right\} + \frac{\beta^k}{N}\partial_i\mathcal{A}_k, && \text{(B.9)} \\ F_{ij} &= \partial_i\mathcal{A}_j - \partial_j\mathcal{A}_i. && \text{(B.10)} \end{aligned}$$

All components of the stress-energy tensor are given by taking the variation of the effective action w.r.t. N, β^i and q_{ij}. The result is

$$\begin{aligned} T_{\perp\perp} &\equiv -\frac{1}{\sqrt{q}}\frac{\delta}{\delta N}\int dt d^3x N\sqrt{q}\mathcal{L} \\ &= \frac{M^2}{g^2}\left\{\frac{1}{2}q^{ij}F_{\perp i}F_{\perp j} - \left[1 + \frac{1}{2}\left(q^{kl} + \frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\frac{1}{\sqrt{q}}\partial_i(\sqrt{q}q^{ij}F_{\perp j}) + \frac{1}{4}q^{ik}q^{jl}F_{ij}F_{kl}\right\} \\ &\quad + \alpha M^2\left\{-\frac{1}{2}(\nabla^\mu D_\mu\phi)^2 + \frac{\beta^i\mathcal{A}_i}{N}\left(1 + \frac{\beta^j\mathcal{A}_j}{N}\right)\partial_\perp(\nabla^\mu D_\mu\phi) - q^{ij}\mathcal{A}_i\partial_j(\nabla^\mu D_\mu\phi)\right\}, \\ T_{\perp i} &\equiv -\frac{1}{\sqrt{q}}\frac{\delta}{\delta\beta^i}\int dt d^3x N\sqrt{q}\mathcal{L} \\ &= \frac{M^2}{g^2}\left[q^{jk}F_{ij}F_{\perp k} + \frac{\beta^j}{N}\mathcal{A}_i\mathcal{A}_j\frac{1}{\sqrt{q}}\partial_k(\sqrt{q}q^{kl}F_{\perp l})\right] \end{aligned}$$

$$
\begin{aligned}
&+\alpha M^2\left\{-\left(1+\frac{\beta^j\mathcal{A}_j}{N}\right)\mathcal{A}_i\partial_\perp(\nabla^\mu D_\mu\phi)\right.\\
&\left.-\left[1-\frac{\beta^j\mathcal{A}_j}{N}+\frac{1}{2}\left(q^{kl}-\frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right]\partial_i(\nabla^\mu D_\mu\phi)\right\},\\
T^{ij}\equiv&-\frac{1}{\sqrt{q}}\frac{\delta}{\delta q_{ij}}\int dtd^3xN\sqrt{q}\mathcal{L}\\
=&\frac{M^2}{g^2}\left\{\left(\frac{1}{2}q^{kl}F_{\perp k}F_{\perp l}-\frac{1}{4}q^{km}q^{ln}F_{kl}F_{mn}\right)q^{ij}-q^{ik}q^{jl}F_{\perp k}F_{\perp l}+q^{ik}q^{jl}q^{mn}F_{km}F_{ln}\right.\\
&\left.-q^{ik}q^{jl}\mathcal{A}_k\mathcal{A}_l\frac{1}{\sqrt{q}}\partial_m(\sqrt{q}q^{mn}F_{\perp n})\right\}\\
&+\alpha M^2\left\{\left[\frac{1}{2}(\nabla^\mu D_\mu\phi)^2\right.\right.\\
&\left.-\left(1-\frac{\beta^k\mathcal{A}_k}{N}+\frac{1}{2}\left(q^{kl}-\frac{\beta^k\beta^l}{N^2}\right)\mathcal{A}_k\mathcal{A}_l\right)\partial_\perp(\nabla^\mu D_\mu\phi)+q^{kl}\mathcal{A}_k\partial_l(\nabla^\mu D_\mu\phi)\right]q^{ij}\\
&\left.+q^{ik}q^{jl}\mathcal{A}_k\mathcal{A}_l\partial_\perp(\nabla^\mu D_\mu\phi)-q^{ik}q^{jl}(\mathcal{A}_k\partial_l+\mathcal{A}_l\partial_k)(\nabla^\mu D_\mu\phi)\right\}.
\end{aligned}
\tag{B.11}
$$

The Einstein equation is

$$
M_{pl}^2G_{\mu\nu}u^\mu u^\nu=T_{\perp\perp},\quad M_{pl}^2G_{\mu i}u^\mu=T_{\perp i},\quad M_{pl}^2G^{ij}=T^{ij},
\tag{B.12}
$$

where

$$
u^\mu=\left(\frac{\partial}{\partial t}\right)^\mu-\beta^i\left(\frac{\partial}{\partial x^i}\right)^\mu.
\tag{B.13}
$$

Just for completeness, the equation of motion for $\mathcal{A}_i$ is

$$
\begin{aligned}
&\frac{1}{g^2}\left\{\frac{1}{\sqrt{q}}\partial_\perp(\sqrt{q}q^{ij}F_{\perp j})-\left[\left(q^{ij}-\frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_j-\frac{\beta^i}{N}\right]\frac{1}{\sqrt{q}}\partial_k(\sqrt{q}q^{kl}F_{\perp l})\right.\\
&\left.+\frac{1}{N\sqrt{q}}\partial_j(N\sqrt{q}q^{ik}q^{jl}F_{kl})+\left(q^{kj}F_{\perp j}\frac{\partial_k\beta^i}{N}-q^{ij}F_{\perp j}\frac{\partial_k\beta^k}{N}\right)\right\}\\
&+\alpha\left\{\left[\left(q^{ij}-\frac{\beta^i\beta^j}{N^2}\right)\mathcal{A}_j-\frac{\beta^i}{N}\right]\partial_\perp(\nabla^\mu D_\mu\phi)-q^{ij}\partial_j(\nabla^\mu D_\mu\phi)\right\}=0.
\end{aligned}
\tag{B.14}
$$

C Static Post-Newtonian Gravity

The PPN parameters $\beta_{\rm PPN}$ and $\gamma_{\rm PPN}$ are defined in the $1/r$-expansion of a spherically symmetric, static metric in the isotropic coordinate as

$$
ds^2=-\left[1-\frac{2m}{r}+\frac{2\beta_{\rm PPN}m^2}{r^2}+O(m^3/r^3)\right]dt^2+\left[1+\frac{2\gamma_{\rm PPN}m}{r}+O(m^2/r^2)\right]\left(dr^2+r^2d\Omega_2^2\right).
\tag{C.1}
$$

In this coordinate, $\beta_{\rm PPN}$ and $\gamma_{\rm PPN}$, respectively, measure the amount of non-linearity and the amount of space curvature produced by a mass. The values in General Relativity is $\beta_{PPN} = \gamma_{PPN} = 1$. Experimental limits on them are

$$|\beta_{\rm PPN} - 1| < 10^{-3}, \quad |\gamma_{\rm PPN} - 1| < 10^{-3}. \tag{C.2}$$

In this section we calculate $\beta_{\rm PPN}$ and $\gamma_{\rm PPN}$ for the gauged ghost condensate.

For the isotropic coordinate

$$ds^2 = -N(r)^2 dt^2 + B(r)^2 \left(dr^2 + r^2 d\Omega_2^2\right), \tag{C.3}$$

Einstein tensor is given by

$$\begin{aligned}
G_{\perp\perp} &= \frac{1}{B^2}\left[\left(\frac{B'}{B}\right)^2 - \frac{4B'}{rB} - \frac{2B''}{B}\right],\\
G_{\perp r} &= G_{\perp\theta} = 0,\\
G^{rr} &= \frac{1}{B^4}\left[2\frac{B'}{B}\frac{N'}{N} + \left(\frac{B'}{B}\right)^2 + \frac{2}{r}\frac{B'}{B} + \frac{2}{r}\frac{N'}{N}\right],\\
G^{\theta\theta} &= \frac{1}{r^2 B^4}\left[\frac{N''}{N} + \frac{N'}{rN} + \frac{B'}{rB} + \frac{B''}{B} - \left(\frac{B'}{B}\right)^2\right],\\
G^{r\theta} &= 0.
\end{aligned} \tag{C.4}$$

We shall apply the formulae obtained in Appendix B to this metric by setting

$$N = N(r), \quad \beta^i = 0, \quad q_{ij}dx^i dx^j = B(r)^2\left(dr^2 + r^2 d\Omega_2^2\right), \quad A_i dx^i = A(r)dr. \tag{C.5}$$

The result is

$$\begin{aligned}
M^{-2}T_{\perp\perp} &= \frac{1}{g^2}\left[\frac{1}{2}\left(\frac{F_\perp}{B}\right)^2 - \left(1 + \frac{A^2}{2B^2}\right)\frac{(r^2 B F_\perp)'}{r^2 B^3}\right] - \alpha\left[\frac{1}{2}(dA)^2 + \frac{A(dA)'}{B^2}\right],\\
M^{-2}T_{\perp r} &= -\alpha\left(1 + \frac{A^2}{2B^2}\right)(dA)',\\
M^{-2}T_{\perp\theta} &= 0,\\
M^{-2}T^{rr} &= \frac{1}{B^2}\left\{-\frac{1}{g^2}\left[\frac{1}{2}\left(\frac{F_\perp}{B}\right)^2 + \frac{A^2(r^2 B F_\perp)'}{r^2 B^5}\right] + \alpha\left[\frac{1}{2}(dA)^2 - \frac{A(dA)'}{B^2}\right]\right\},\\
M^{-2}T^{\theta\theta} &= \frac{1}{r^2 B^2}\left\{\frac{1}{2g^2}\left(\frac{F_\perp}{B}\right)^2 + \alpha\left[\frac{1}{2}(dA)^2 + \frac{A(dA)'}{B^2}\right]\right\},\\
M^{-2}T^{r\theta} &= 0,
\end{aligned} \tag{C.6}$$

where

$$\begin{aligned}
F_\perp &= -\frac{1}{N}\left[N\left(1 + \frac{A^2}{2B^2}\right)\right]',\\
dA &= \frac{(r^2 NBA)'}{r^2 NB^3}.
\end{aligned} \tag{C.7}$$

In order to calculate $\beta_{\rm PPN}$ and $\gamma_{\rm PPN}$, we expand all variables by $1/r$,

$$
\begin{aligned}
N(r) &= 1 + \sum_{n=1}^{\infty} \frac{N_n}{r^n}, \\
B(r) &= 1 + \sum_{n=1}^{\infty} \frac{B_n}{r^n}, \\
A(r) &= \sum_{n=0}^{\infty} \frac{A_n}{r^n}.
\end{aligned} \tag{C.8}
$$

Accordingly, we obtain the $1/r$-expansion of the Einstein equation.

The $(\perp r)$-component of the Einstein equation is $\alpha(dA)' = 0$, which with the above $1/r$-expansion implies that $A = 0$. In this case non-vanishing components of the stress energy tensor are

$$
\begin{aligned}
M^{-2}T_{\perp\perp} &= \frac{1}{g^2}\left[\frac{1}{2}\left(\frac{N'}{NB}\right)^2 + \left(1+\frac{A^2}{2B^2}\right)\frac{1}{r^2B^3}\left(\frac{r^2BN'}{N}\right)'\right], \\
B^2M^{-2}T^{rr} &= -\frac{1}{g^2}\left[\frac{1}{2}\left(\frac{N'}{NB}\right)^2 + \frac{A^2}{r^2B^5}\left(\frac{r^2BN'}{N}\right)'\right], \\
r^2B^{-2}M^{-2}T^{\theta\theta} &= \frac{1}{2g^2}\left(\frac{N'}{NB}\right)^2.
\end{aligned} \tag{C.9}
$$

The leading term in the (rr)-component of the Einstein equation says that

$$
B_1 = -N_1. \tag{C.10}
$$

With this relation, the leading term in $G_{\perp\perp} - B^2G^{rr} = \kappa^2(T_{\perp\perp} - B^2T^{rr})$ is

$$
(M^2 - 2g^2M_{pl}^2)\left(N_2 - \frac{N_1^2}{2}\right) = 0. \tag{C.11}
$$

Hence, unless $M^2 = 2g^2M_{pl}^2$, we obtain

$$
N_2 = \frac{N_1^2}{2}. \tag{C.12}
$$

¿From (C.10) and (C.12) we obtain

$$
\beta_{\rm PPN} = \gamma_{\rm PPN} = 1. \tag{C.13}
$$

Therefore, there is no constraint from the spherically symmetric, static post-Newtonian gravity. The leading correction to General Relativity appears in B_2/N_1^2. Actually, the leading term in the $(\perp\perp)$-component of the Einstein equation says that

$$
\frac{B_2}{N_1^2} = \frac{1}{4}\left(1 + \frac{M^2}{2g^2M_{pl}^2}\right), \tag{C.14}
$$

and this value of B_2/N_1^2 has a deviation from the GR value 1/4. However, this deviation is in the post-post Newtonian order and beyond the current ability of gravity experiments.

D A Simple Way to Calculate the Energy Loss in Čerenkov Radiation

Consider a lagrangian in the presence of an external source,

$$\mathcal{L}_{\text{Total}} = \mathcal{L}_\phi(\phi, \partial\phi) + \mathcal{L}_S(\phi, S), \tag{D.1}$$

where $_S$ represent the source term *e.g.*, $\mathcal{L}_S = S\phi$. If the action without the source $\mathcal{S}_\phi = \int d^4x \mathcal{L}_\phi$ is invariant under the transformation $\phi \to \phi' = \phi + \delta\phi$, there is an associated conserved Noether current,

$$J^\mu_\phi = \frac{\delta \mathcal{L}_\phi}{\delta(\partial_\mu \phi)}\delta\phi - V^\mu, \tag{D.2}$$

with V^μ given by

$$\delta\mathcal{L}_\phi = \partial_\mu V^\mu. \tag{D.3}$$

The presence of the (fixed) external source "breaks" the invariance of the transformation, so the current is no longer conserved,

$$\partial_\mu J^\mu_\phi = \delta\mathcal{L}_S. \tag{D.4}$$

Integrating over space, we obtain

$$\int d^3x \delta\mathcal{L}_S = \int d^3x\, \partial_\mu J^\mu_\phi = \partial_0 Q_\phi + \oint J^i dS_i. \tag{D.5}$$

The right hand side is just the total charge flowing into the ϕ field and infinity, so it must be compensated by the loss of the total charge of the source, $-dQ_S/dt$. For energy, the corresponding symmetry transformation is the time translation, so $\delta\phi = \dot{\phi}$. The energy loss for a source $\mathcal{L}_S = S\phi$ is then simply given by

$$\frac{dE_S}{dt} = -\int d^3x\, S\, \dot{\phi}. \tag{D.6}$$

In the linearized approximation, $\dot{\phi}$ can be easily solved in terms of a given source in momentum space. It is then straightforward to compute the energy loss using Eq. (D.6).

E A Null Test for a Spin-Dependent Force Law

In (ungauged) ghost condensation and in electromagnetism, there is a potential between spins of the form

$$V_n(r) = c_n^2(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla}) r^n, \tag{E.1}$$

where $n = 1$ for the $1/r$ spin-spin potential in ghost condensation, and $n = -1$ for the $1/r^3$ magnetic dipole-dipole potential. Equivalently, these theories can be described by the non-relativistic interaction

$$\mathcal{L}_{\text{int}} = c_n \vec{s} \cdot \vec{\nabla} \pi_n, \tag{E.2}$$

where the $\pi_n\pi_n$ correlator is

$$\langle \pi_n(\vec{x})\pi_n(0)\rangle = |\vec{x}|^n. \tag{E.3}$$

Doing an integration by parts on Eq. (E.2), we see that π_n is sourced by the divergence of $\vec{s}$, so for a divergence-free spin configuration, the π_n field is zero away from the source.

One interesting divergence-free spin configuration is a ring of spin. This could be established with a toroidal electromagnet, and as is well-known, the magnetic field is zero outside of the torus. In the case of (ungaged) ghost condensation, the Goldstone field would also be zero outside of torus, so this geometry is not well-suited to study the long-range inverse-square law force from Ref. [5]. However, the potential in Eq. (7.25) is not of the form of Eq. (E.1). In particular, the transverse modes of $\vec{A}_i$ couple directly to $\vec{s}$ and not to its divergence.

Consider the following setup, where a ring of radius R with net spin S_1 is separated a distance r away from a point spin S_2.

$$INSERTIMAGEHERE \tag{E.4}$$

It is straightforward to calculate the potential from Eq. (7.25):

$$V(r) = \frac{M^2 g^2}{F^2}\frac{S_1 S_{2y}}{8\pi r}\frac{R}{r} + \mathcal{O}(R^3/r^3), \tag{E.5}$$

where S_{2y} is the projection of S_2 in the vertical direction. Note that this potential does not depend on α, because α only controls the coupling of the longitudinal mode of $\vec{A}$. Though this potential goes as $1/r^2$, there is in principle no magnetic field leakage from the ring S_1, so it should be easier to design a null experiment to look for this long-range spin-spin potential.

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